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ANALYSIS OF INDUCTION MOTOR DRIVEN BY RECTANGULAR WAVE CURRENT

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I. INTRODUCTION

In inverters used for induction motors, there are two types: self commutated inverters with impressed voltage in a DC link which supply rectangular wave voltage as the voltage source and those with impressed current which supply rectangular wave current as the current source.

As is well known, the generated torque in the motor is pulsated because the current supplied to the motor into rectangular wave current.

In this article, a three-phase induction motor driven by a 6-pulse impressed current type inverter will be treated and comparisons were made with measured values and computed values refer to torque. In the analysis of the induction motor, the *d–q* axis method generally used in the analysis of synchronous motors was employed for simplification. The computation was handled by the modern control theory and the convenient state transition theory was employed in numerical value computation using a digital computer.

II. SELF INVERTER WITH IMPRESSED CURRENT IN A DC LINK

Fig. 1 shows the circuit diagram of the self-commutated inverter with impressed current in a DC link. In this figure, $Th_1 \sim Th_6$ are the main thyristors, $S_1 \sim S_6$ are the turn-off thyristors, and $C_1 \sim C_6$ are capacitors for commutation. The reactor L_d in the DC link is to impress the DC current I_d and since the inverter serves as the current source, the reactor is required in this type of inverter. (An impressed voltage type inverter can be formed if a capacitor is inserted in the DC link instead of the reactor L_d).

The operation of commutation from *R* phase to *S* phase is explained by reference to *Fig. 1*. At that time C_1 is charged at the polarity shown in the diagram. Commutation is initiated by the firing of turn-off thyristor S_1 and simultaneous firing of main thyristor Th_2, then the dischage current from C_1 flows $C_1 \rightarrow Th_1 \rightarrow S_1$ and Th_1 is turned off in a moment. This causes current I_d to be commutated in S_1 and C_1 is discharged at a constant current by I_d.

At this point Th_2 has a reverse bias and not be turned on. The polarity of C_1 is reversed and is gradually charged in the reverse polarity. The reverse bias disappears and Th_2 is turned on. Then I_d starts to be divided between S_1 and Th_2, the current of S_1 is reduced and the Th_2 current increases. Then the total I_d current is transferred to Th_2 and commutation is completed.

By repeating such commutation, rectangular wave current with a conduction of about 120° is supplied to the load. *Fig. 2* shows the waveforms of the phase voltage, line current and commutating capacitor voltage when an induction motor is used as the load. There are cases depending on the application when the charging circuit is not necessary because of fixed commutation.

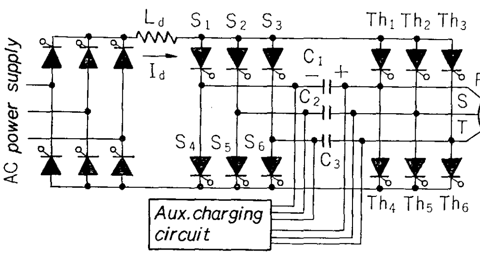
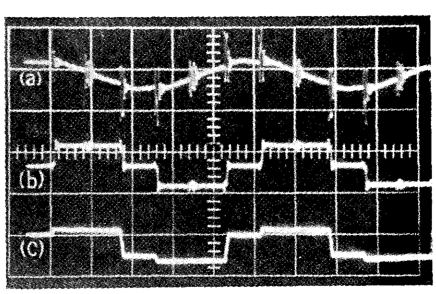


Fig. 1 Circuit diagram of inverter with impressed current in DC link



(a) Phase voltage 50 V/DIV (T=200 ms)
(b) Line current 20 A/DIV
(c) Capacitor voltage 400 V/DIV

Fig. 2 Wave forms of current and voltage

III. CIRCUIT EQUATION OF INDUCUION MOTOR

Fig. 3 shows the circuit of the inverter-induction motor (three phase, two pole) system described in this article. The inverter-induction motor system circuit shown in *Fig. 3* is difficult to analyze as things stand. Therefore, an equivalent circuit convenient for analysis is introduced.

First, the 3-phase windings a, b and c converted to two orthogonal d–q windings by means of the two axis theory (d–q coordinate system)(1). The positions of the d–q axes can be selected optionally but in this case, the d axis as shown above the stator windings in *Fig. 3* was selected to be delayed 30° from the a–phase winding. In this case, the transformation matrix $[C_1]$ in respect to the stator windings becomes as follows:

$$[C_1]=\sqrt{\frac{2}{3}}\;\begin{array}{c|c|c|} & i_{1d} & i_{1q} \\ \hline i_{1a} & \sqrt{3}/2 & 1/2 \\ \hline i_{1b} & -\sqrt{3}/2 & 1/2 \\ \hline i_{1c} & & -1 \\ \hline \end{array} \quad \cdots\cdots\cdots\cdots (1)$$

The transformation matrix $[C_2]$ in respect to the rotor windings is a rotating matrix since the winding rotates in respect to the d–q axes. Then C_2 is as *eq* (2).

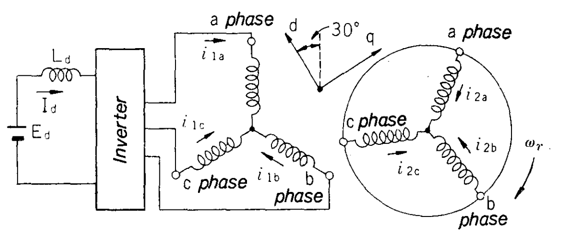


Fig. 3 Inverter-induction motor system

$$[C_2]=\sqrt{\frac{2}{3}}\;\begin{array}{c|c|c|} & i_{2d} & i_{2q} \\ \hline i_{1a} & \cos\theta_\alpha & \sin\theta_\alpha \\ \hline i_{2b} & \cos\theta_\beta & \sin\theta_\beta \\ \hline i_{2c} & \cos\theta_\gamma & \sin\theta_\gamma \\ \hline \end{array} \quad \cdots\cdots\cdots\cdots (2)$$

where

$$\theta_\alpha=\omega_r t+\delta,\ \theta_\beta=\theta_\alpha+\frac{2}{3}\pi,\ \theta_\gamma=\theta_\alpha-\frac{2}{3}\pi$$

ω_r = angular velocity

where δ is the angle between the d axis and the rotor a–phase winding $t=0$. This angle corresponds to the load angle in synchronous motors and is constant under steady state although it changes in induction motors because of slip.

The currents i_{1a}, i_{1b} i_{1c} which flow in the stator windings can be transformed into the d–q axis components i_{1d} and i_{1q} by means of equation (1).

$$\begin{bmatrix} i_{1d} \\ i_{1q} \end{bmatrix}=\sqrt{\frac{2}{3}}\,[C_1]_t\begin{bmatrix} i_{1a} \\ i_{1b} \\ i_{1c} \end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix} \frac{\sqrt{3}}{2}i_{1a}-\frac{\sqrt{3}}{2}i_{1b} \\ \frac{1}{2}i_{1a}+\frac{1}{2}i_{1b}-i_{1c} \end{bmatrix} \quad \cdots (3)$$

The currents in the rotor can also be divided into the some components using equation (2).

Fig. 4 shows the induction motor circuit using the d-q axes. In this case:

$$\left.\begin{aligned} L_1&=l_{1a}+\frac{3}{2}L_{a1} \\ L_2&=l_{a2}+\frac{3}{2}L_{a2} \\ M&=\frac{3}{2}M_a \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (4)$$

where l_{a1} and l_{a2} are the leakage reactances of one phase component of the stator windings respectively. L_{a1} and L_{a2} are the effective reactances and R_1 and R_2 are the resistances of the same component of the windings. M_a is the mutual inductance between the stator and rotor windings of one phase.

The equations are as following for the circuit in *Fig. 4* when the current and voltage vectors $[I]$ and $[V]$ respectively are:

$$\left.\begin{aligned} [I]&=[i_{1d},\ i_{1q},\ i_{2d},\ i_{2q}]_t \\ [V]&=[v_{1d},\ v_{1q},\ 0,\ 0]_t \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (5)$$

then,

$$[V]=[Z]\times[I] \quad \cdots\cdots\cdots\cdots (6)$$

where $[Z]$ is the impedance matrix. From the circuit shown in *Fig. 4*, the following can be obtained directly:

$$[Z]=\begin{array}{c|c|c|c|c|} & 1d & 1q & 2d & 2q \\ \hline 1d & R_1+L_1P & & MP & \\ \hline 1q & & R_1+L_1P & & MP \\ \hline 2d & MP & \omega_r M & R_2+L_2P & \omega_r L_2 \\ \hline 2q & -\omega_r M & MP & -\omega_r L_2 & R_2+L_2P \\ \hline \end{array}$$

The following is obtained when equation (6) is divided into the positive phase sequence component

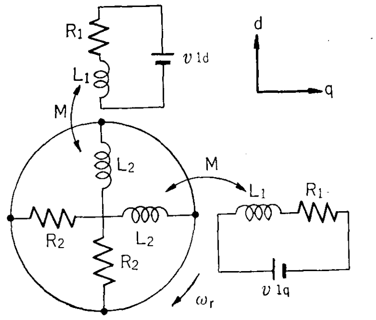


Fig. 4 Representation of induction motor using d-q axes

and the negative phase sequence component by the method of symmetrical coordinates:

$$\begin{bmatrix} v_{1f} \\ v_{1b} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+L_1P & & MP & \\ & R_1+L_1P & & MP \\ M(P-j\omega_r) & & R_2+L_2\times(P-j\omega_r) & \\ & M(P+j\omega_r) & & R_2+L_2\times(P+j\omega_r) \end{bmatrix} \times \begin{bmatrix} i_{1f} \\ i_{1b} \\ i_{2f} \\ i_{2b} \end{bmatrix} \quad \cdots\cdots (7)$$

where

$$v_{1f}=(v_{1d}+jv_{1q})/\sqrt{2},\ v_{1b}=(v_{1d}-jv_{1q})/\sqrt{2}$$
$$i_{1f}=(i_{1d}+ji_{1q})/\sqrt{2},\ i_{1b}=(i_{1d}-ji_{1q})/\sqrt{2}$$
$$i_{2f}=(i_{2d}+ji_{2q})/\sqrt{2},\ i_{2b}=(i_{2d}-ji_{2q})/\sqrt{2}$$

It is clear from equation (7) that the negative phase sequence component is the conjugate of the postive phase sequence component and it is sufficient to analyze only the positive phase sequence component in equation (7).

From the above, the following equation can be obtained for the induction motor circuit as shown in *Fig. 4*.

$$\begin{bmatrix} v_{1f} \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+L_1P & MP \\ M(P-j\omega_r) & R_2+L_2(P-j\omega_r) \end{bmatrix} \times \begin{bmatrix} i_{1f} \\ i_{2f} \end{bmatrix} \quad \cdots\cdots (8)$$

IV. MEASUREMENTS FROM CORODINATED WHICH ROTATE WITH COMMUTATION

Fig. 5 shows an ideal waveform of the output current of the self-commutated inverter with impressed current in a DC link. The resultant vector i_f ($i_{1f}=\{i_{1d}+ji_{1q}\}/\sqrt{2}$) of i_{1d} and i_{1q} which are the three phase current with equation (3) becomes transposed a constant vector and rotates 60° in steps every time commutation occurs. It has 6 states as are shown in *Fig. 6*.

In this case, the concept that the coordinates rotates 60° in steps in relation to the d-q coordinates every time commutation occurs is named as the $\alpha-\beta$ coordinates. The relation between the $\alpha-\beta$ axes and the i_{1f} vector measured from the d-q coordinates, is as shown in *Fig. 7* with the optional 60° spacing.

The i_{1f} vector and the $\alpha-\beta$ axes just prior to commutation are in the positions i_{1f} (mT) and $\alpha_0-\beta_0$ respectively. When commutation occurs, i_{1f} first rotates from the i_{1f} (mT) position to the i_{1f} (mT_+) position. In this way, i_{1f} rotates 60° in respect to the $\alpha-\beta$ axes. Then the $\alpha-\beta$ axes rotate 60° from α_0-β_0 to α_+-β_+ and the relation between i_{1f} and the $\alpha-\beta$ axes returns to its original position just before commutation. This is repeated every time commutation occurs and the relation between i_{1f} and the α-β axes can show two states in any 60° spacing.

Now the circuit equation of the induction motor in respect to the $\alpha-\beta$ coordinates will be considered as same as the d-q coordinaties. In respect to the $\alpha-\beta$ coordinates, when the current vector $[I']$ and the voltage vector $[V']$ are

$$\left.\begin{aligned} [I'] &= [i_{1\alpha},\ i_{1\beta},\ i_{2\alpha},\ i_{2\beta}]_t \\ [V'] &= [v_{1\alpha},\ v_{1\beta},\ 0,\ 0]_t \end{aligned}\right\} \quad \cdots\cdots (9)$$

and the positive phase sequence component which is separated by means of the method of symmetrical coordinates are

$$i_{1F}=(i_{1\alpha}+ji_{1\beta})/\sqrt{2} \qquad i_{2F}=(i_{2\alpha}+ji_{2\beta})/\sqrt{2}$$
$$v_{1F}=(v_{1\alpha}+jv_{1\beta})/\sqrt{2}$$

The condition in *Fig. 8* is obtained from the relation between the i_{1f} vector and the $\alpha-\beta$ axes in *Fig. 7*. In *Fig. 8*, the i_{1F} vector is held in the i_{1F} (mT)

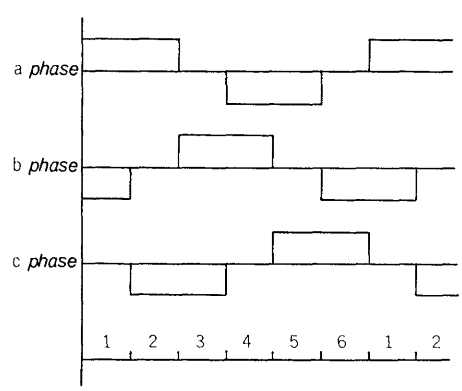


Fig. 5 Waveform of ideal self-commutated inverter with impressed current in a DC link

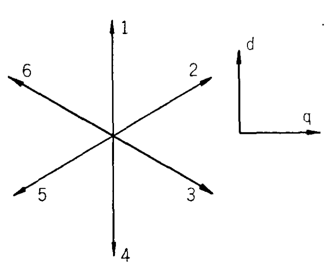


Fig. 6 Stator current vectors in d-q plane

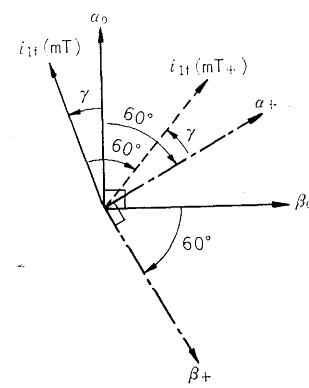


Fig. 7 Relation between stator current vectors and $\alpha-\beta$ axes

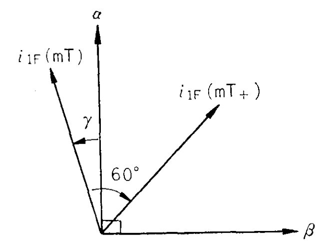


Fig. 8 **Stator current vectors in α-β plane**

condition except during commutation and is shifted to $i_{1F}\,(mT_+)$ only for a short time during commuation. The relation between $i_{1F}\,(mT)$ and $i_{1F}\,(mT_+)$ is a follows:

$$i_{1F}\,(mT_+)=e^{j\varphi}\times i_{1F}\,(mT) \quad\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

where $\varphi=60°$

From the above, the circuit equation of the induction motor in respect to the α-β coordinates becomes as follows. First, the α–β axes remain stationary in respect to the d–q axes except during commutation and the i_{1F} vector is also constant in the $i_{1F}\,(mT)$ position. Tnerefore, the impedance matrix $[Z']$ has the same relation as $[Z]$ and is follow from equation (8):

$$\begin{bmatrix} v_{1F} \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+L_1P & MP \\ M(P-j\omega_r) & R_2+L_2(P-j\omega_r) \end{bmatrix} \times \begin{bmatrix} i_{1F} \\ i_{2F} \end{bmatrix} \quad\cdots\cdots\cdots (11)$$

During commutation, there is a shift from $i_{1F}\,(mT)$ to $i_{1F}(mT_+)$ when commutation first begins. Therefore, i_{2F} changs from $i_{2F}\,(mT)$ to $i_{2F}\,(mT_+)$. At this time, the α–β coordinates are stationary in respect to the d–q coordinates so that the relation of i_{2F} is obtained by integration by equation (11) from the time mT just before commutation to the time mT_+ just after commutation. The commutation occurs for only a short time in this integrations. Comparing with differential terms, the other terms are small and can be neglected. Therefore, from equation (11);

$$M\{i_{1F}\,(mT_+)-i_{1F}\,(mT)\}+L_2\{i_{2F}\,(mT_+)-i_{2F}\,(mT)\}=0 \quad\cdots\cdots\cdots (12)$$

When equation (12) is substitued in equation (10), the following is obtained:

$$i_{2F}\,(mT_+)=\frac{M}{L_2}(1-e^{j\varphi})\times i_{1F}(mT)+i_{2F}(mT) \quad\cdots (13)$$

When commutation is completed, the α–β coordinates are rotated 60° so that i_{2F} is shifted to the $i_{2F}(mT'_+)$ position rotated 60° from the $i_{2F}(mT_+)$ position. This relation can be shown by the following equation as a projection to the axes rotated 60° as in the case of i_{1F}. However, at this time, the rotational direction is opposite to that of i_{1F}.

$$\begin{aligned} i_{2F}\,(mT_+')&=e^{-j\varphi}\times i_{2F}\,(mT_+) \\ &=\frac{M}{L_2}(e^{-j\varphi}-1)\times i_{1F}(mT)+e^{-j\varphi}\times i_{2F}(mT) \end{aligned} \quad\cdots\cdots\cdots (14)$$

At the time of commutation, the i_{1F} vector is changed once from $i_{1F}\,(mT)$ to $i_{1F}\,(mT_+)$ but in ideal inverter with impressed current in a DC link, this commutation time can be neglected. Since the axis rotation occurs just after commutation, the time for which the position is $i_{1F}\,(mT_+)$ can also be neglected. Therefore, the i_{1F} vector can be considered as constant during commutation. The same concepts can also be applied for the i_{2F} vector. The time in the position $i_{2F}\,(mT_+)$ after the change from i_{2F} which occurs once can be neglected and just after commutation, the change can be considered as directly from $i_{2F}\,(mT)$ to $i_{2F}\,(mT'_+)$. Therefore, at the time of commutation, the following hold true:

$$\left.\begin{aligned} i_{1F}(nT_+)&=i_{1F}(nT) \\ i_{2F}(nT_+)&=\frac{M}{L_2}(e^{-j\varphi}-1)\times i_{1F}(nT)+e^{-j\varphi}\times i_{2F}(nT) \end{aligned}\right\} \quad\cdots\cdots\cdots (15)$$

where nT is the time just before commutation and nT_+ is the time just after commutation.

V. ANALYSIS BY THE STATE TRANSITION METHOD

Generally when electrical circuit equations were used, analysis was by means of the Laplace transformation. The circuits equations (11) and (15) can also be resolved by Laplace transformation. However, in this case the analysis was performed by the state transition method using modern control theory.

When the inverter frequency $(f=1/6T)$ and the angular velocity of the rotor ω_r are constant, the following can be obtained from equation (11):

$$\left.\begin{aligned} Pi_{1F}&=0 \\ Pi_{2F}&=j\omega_r\frac{M}{L_2}i_{1F}+\left(j\omega_r-\frac{R_2}{L_2}\right)i_{2F} \end{aligned}\right\} \quad\cdots\cdots\cdots (16)$$

In this case, if the state variable X expressed as $X=[i_{1F},\, i_{2F}]_t$ equation (16) can be expressed as follows:

$$\dot{X}=A\cdot X \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (17)$$

where A is as follows:

$$A=\begin{bmatrix} & \\ j\omega_r M/L_2 & j\omega_r-R_2/L_2 \end{bmatrix}$$

The relation shown in equation (15) at the time of commutation can be expressed as follows:

$$X(nT_+)=B\cdot X(nT) \quad \cdots\cdots (18)$$

where B is as follows:

$$B=\begin{bmatrix} 1 & \\ (e^{-j\varphi}-1)\times M/L_2 & e^{+j\varphi} \end{bmatrix}$$

In this case, when $\Phi(t)=e^{At}$, $H(t)=\Phi(t)\cdot B$, equations (17) and (18) can be solved as follows: [2]

$$X(t)=H(t-nT)\cdot X(nT) \quad \cdots\cdots (19)$$

From this the value of X (in this case, the i_{1F}, i_{2F} value) can be obtained in respect to any point in time.

The cuurent i_{1F}, i_{2F} in the steady state ($n\to\infty$) can be obtained from equation (19) by making n integers starting from zero. But in this case, the steady state value of $X(nT)$ is obtained in this way. When $X(nT)$ is in the steady state, then:

$$X(n+\bar{1}T)=X(nT) \quad \cdots\cdots (20)$$

When calculating the current in the steady state using the above, the following is employed:

$$i_{2F}(nT_\infty)=C_0/(1-D_0)\times i_{1F} \quad \cdots\cdots (21)$$

where

$$C_0=\frac{a}{b}(e^{bT}-1)+e^{bT}(e^{-j\varphi}-1)\cdot\frac{M}{L_2}, \quad D_0=e^{bT}\cdot e^{-j\varphi}$$

$$a=j\omega_r M/L_2, \qquad b=j\omega_r-R_2/L_2, \qquad \varphi=60°$$

From the above, the current at any point in time can be obtained by substituting the final value of $X(nT)$ obtained from equation (21) in equation (19):

$$\left.\begin{array}{l} i_{1F}(t)=\text{constant} \\ i_F(t)=C\cdot i_{1F}+D\cdot i_{2F}(nT_\infty) \end{array}\right\} \cdots\cdots (22)$$

where C and D are value when t is used for T at C_0 and D_0.

When the output torque T is expressed in terms of $[G]$ ($[G]$ is the coefficient of the angular velocity in the impedance matrix) the following is obtained[3]:

$$T=[I]_t^*\cdot[G]\cdot[I] \quad \cdots\cdots (23)$$

In this case, $[I]_f^*$ is the conjugate transposed matrix of $[I]$. From equation $[G]$ and $[I]$ are as follows:

$$[G]=\begin{bmatrix} & \\ -jM & -jL_2 \end{bmatrix} \qquad [I]=\begin{bmatrix} i_{1F} \\ i_{2F} \end{bmatrix}$$

Since the output torque is the sum of the positive phase sequence component torque and the negative phase sequence componet torque (since the rotation of the negative component is the reverse of the positive component, the negative component is minus), the following holds true:

$$T=M(i_{1\beta}\cdot i_{2\alpha}-i_{1\alpha}\cdot i_{2\beta}) \quad \cdots\cdots (24)$$

The average torque can be obtained by averaging the instantaneous torque over a spacing of 60°.

VI. EXPANSION TO OPERATION BY THE SINUSOIDAL WAVE CURRENT

The sinusoidal wave can be approximated by a step type waveform with an infinite number of steps. In this case, the output waveform can be made sinusoidally by making the number of pulses infinite in the N pulse inverter. In the 6 pulse inverter, the current vector are divided into six states with the phase defference of 60° as shown in *Fig. 6* and it's direction jumps. In the N pulse inverter, the jump is 360°/N for each commutation and therefore the jump for each case of commutation can be minimized by making N infinite. The current vector rotates smoothly for one cycle.

In equation (21), if the excreme values of $T\to 0$ and $\varphi\to 0$ are considered under $\omega=\varphi/T$ is constant, this is equivalent to driving the induction motor with a sinusoidal wave current[4], and equation (21) shows the secondary current at this time. In this case, φ is the angle which the current vector i_{1f} is moved by one case of commutation and T is the time during one commutation. In the six pulse inverter, $\varphi=60°$ and $T=1/6\times f$ (f is the output frequency).

In equation (22), T and φ are sufficiently small so that e_T and e^φ can be expanded to obtain the following approximate equation (refer to addendum 2):

$$i_{2F} \fallingdotseq \frac{j\omega M}{j\omega L_2+R_2/S}\times i_{1F} \quad \cdots\cdots (25)$$

where $S=(\omega-\omega_r)/\omega$. Equation (25) shows the secondary current when the sinusoidal wave current with a frequency $f(f=\omega/2\pi)$ is flowing in the equivalent circuit of the induction motor shows in *Fig. 9*.

The output torque of the induction motor can be obtained from equation (23) if the primary and secondary currents are known. Therefore, the output torque when the motor is driven by a sinusoidal wave current can be calculated by means of the equivalent circuit as shown in *Fig. 9*. As an example, when the current i_M flowing in the winding M in *Fig. 9* is kept constant (this is equivalent to keeping the flux constant), the torque-speed curves can be obtained.

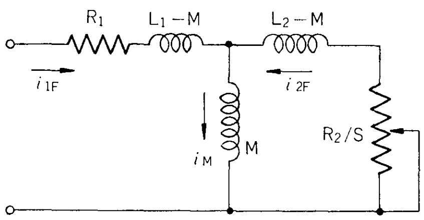


Fig. 9 **Equivalent circuit of induction motor**

When i_M is constant, i_{1F} and i_{2F} are as follows.

$$\left.\begin{aligned} i_{1F} &= \frac{(R_2/S)^2+\omega^2 L_2(L_2-M)+j\omega M R_2/S}{\omega^2(L_2-M)^2+(R_2/S)^2}\times i_M \\ i_{2F} &= -\frac{\omega^2 M(L_2-M)+j\omega M R_2/S}{\omega^2(L_2-M)^2+(R_2/S)^2}\times i_M \end{aligned}\right\} \quad \cdots\cdots (26)$$

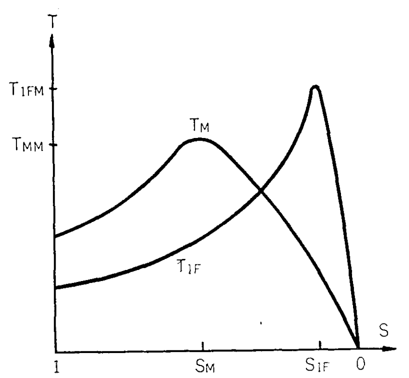
Therefore, the output torque T_M can be obtained as follows by substituting in equation (24):

$$T_M = 2M\times\frac{\omega M R_2/S}{\omega^2(L_2-M^2)+(R_2/S)^2}\times i_M\cdot\bar{i}_M \quad \cdots\cdots (27)$$

This gives a curve with the maximum at $S=R_2/\omega(L_2-M)$. The curves for the torque-speed when the input cuɹrent i_{1F} is constant can be calculated in the same way. The output T_{1F} in this case is as follows:

$$T_{1F} = 2M\times\frac{\omega M R_2/S}{(\omega L_2)^2+(R_2/S)^2}\times i_{1F}\cdot\bar{i}_{1F} \quad \cdots\cdots (28)$$

This gives a curve with the maximum the value at $S=R_2/\omega L_2$. *Fig. 10* shows a typical torque-speed curve when i_M and i_{1F} are constant. In *Fig. 10*, the induction motor is shown to have series characteristics when the primary current i_{1F} is kept constant but this results in over excitation at a light load value (S is nealy equal to 0) since the input current is constant.



$S_M = R_2/\omega(L_2-M)$, $S_{1F}=R_2/\omega L_2$

$T_{MM}=\frac{M^2}{L_2-M}\times i_M\cdot\bar{i}_M$, $T_{1FM}=\frac{M^2}{L_2}\times i_{1F}\cdot\bar{i}_{1F}$

Fig. 10 **Torque speed curve**

VII. COMPARISON OF CALCULATED AND MEASURED VALUES

Fig. 11 shows a block diagram of the test device. A motor with the parameters shown in *Table 1* was used for the test. *Figs. 12* and *13* show the actually measured values and the claculated values for the torque-speed curves. In these figures, the ‘ × ’ marks show the actually measured values and the line curve shows the caclulated values. The point where the torque reaches a maximum in the figures is slightly different in the measured and calculated curves. This is probably caused by errors in measurements of constants.

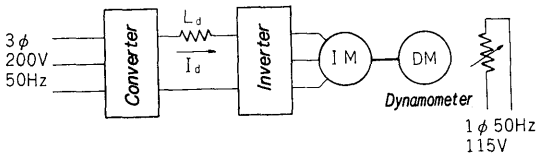


Fig. 11 **Circuit diagram**

Table 1 **Parameters of motor under tested**

Output, No. of poles	0.75 kW, 2 poles
No. of phase, rated voltage	3-phase 200 V (line to line)
R_1	2.78 Ω
R_2	1.71 Ω
$L_1=L_2$	0.2143 H
M	0.2069 H

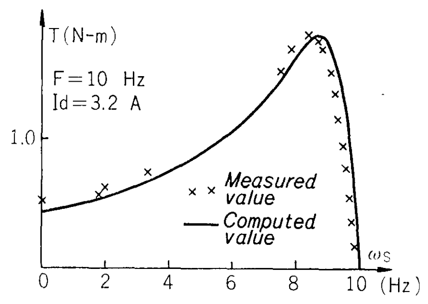


Fig. 12 **Torque-speed curve**

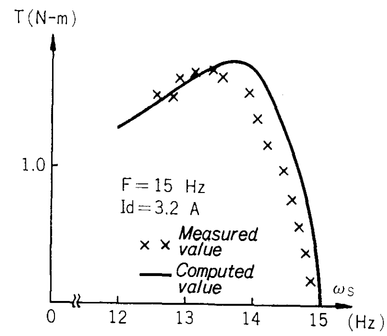


Fig. 13 **Torque-speed curve**

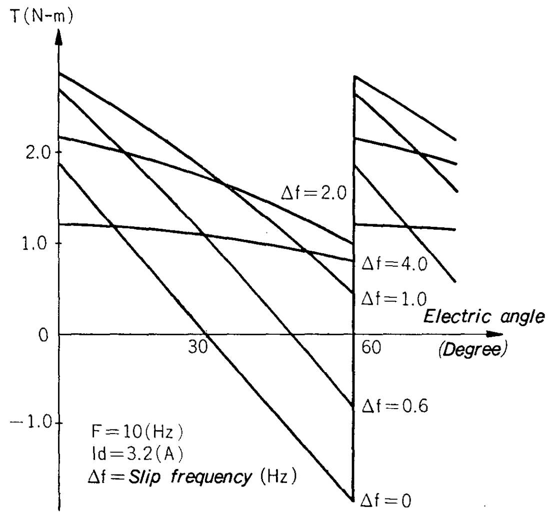


Fig. 14 Computed curves of instantaneous torque

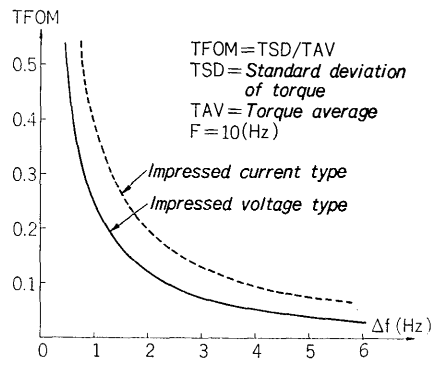


Fig. 15 Comparison of torque pulsation

The torque of an induction motor driven with a sinusoidal wave current can be obtained from equation (28). By using this equation, the torque-speed curve when the r.m.s. value of sinusoidal wave current is $(\sqrt{6}/\pi)\times I_d$ (this is the fundamental r.m.s. value of a rectangular waveform with a conduction about 120° which has a crest value of I_d) can be obtained. This curve agrees with the calculated values when the motor is driven by a rectangular wave current with a crest value of I_d. Because of this, the average value of the output torque of an induction motor driven by a rectangular wave current is almost the same as when the motor is driven by a sinusoidal wave current with the fundamental r.m.s. value of the rectangular wave current with a conduction about 120°. It can also be said that the harmonic current has no relation to the average torque and is related only to the pulsated torque.

Fig. 14 shows the calculated values of the instantaneous torque at various slip frecuencies (the period of this torque is an electrical angle of 60° in the 3-phase, 6 pulse inverter). The instaneous torque values do not depend on the supply frequency when the slip frequencies are the same and are almost equal.

VIII. COMPARISON OF PULSATED TORQUES DRIVEN BY RECTANGULAR WAVE VOLTAGES

In the above case, the amplitude of torque was calculated for an induction motor driven by a square wave current. For reference, a comparison was made with the pulsated torque when the motor is driven by a 3-phase, six pulse rectangular wave voltage. Analysis of drive by means of a rectangular wave voltage has already been described[5] and will be omitted here.

When comparing the pulsated torque, various ideas were used for the coefficient for evaluating the amplitude. For example, there are methods in which the r.m.s. value or the actual values of each harmonic are divided by the mean value. However, in this case, the standard deviation of the torque divided by the mean value was used as the evaluation coefficient. This evaluation coefficient has non dimension and is convenient for comparisons with various types of motors.

Fig. 15 shows an example of a comparison of the pulsated torque for the motor shown in *Table 1.* The evaluation value is almost the same at other frequencies.

Therefore, it is evident that the pulsated torque at the same slip frequency is smoewhat large when the motor is driven by a rectangular wave current.

IX. CONCLUSION

Since the self-commutated inverter with impressed current in a DC link can perform quadrant operation in a simple circuit construction and can perform current control, it has such advantages as safety against commutation failures and will probably come into practical use in the future.

The theories described in this article should make in practical development easier and we are continuing this reasearch in order to decide on the most selective and ideal motor constants and control systems on the basis of the results.

Finally, the authors wish to express their sincere thanks to the Industrial Science Institute of Tokyo University, Associate Professor Fumio Harajima and technical official Takao Koyama for their help and guidance.

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