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HARMONIC ANALYSIS OF CHOPPER CONTROLLED ELECTRIC ROLLING STOCK

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I. INTRODUCTION

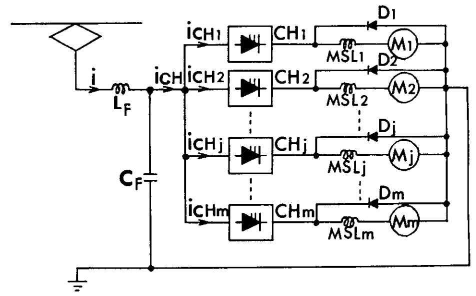
In chopper controlled electric rolling stock, a pulsating current including harmonics flows in the power source line (for example, an overhead power line, hereafter referred to as a power line) for the main operating mechanism of the chopper. This pulsating current can cause inductive interference in communication lines or railway signal lines. In order to prevent such interference, several choppers are generally operated at different phases, and a filter is provided consisting of a current-dividing capacitor and a reactor which blocks the harmonic current on the input side. The analysis of these harmonic problems were already shown by[1],[2] etc. But in such circuits, there are large differences under influences of the number of resultant phases, chopper frequency, conduction rate, and filter resonance frequency when all the choppers operate ideally, as well as when phases are absent or when the conduction rate differs between the choppers. Those abnormal conditions should be considered carefully in practical use.

This paper deals with the inductive interference which occurs in chopper controlled electric rolling stock. The harmonic current and equivalent disturbance current which arise in the multiple-phase chopper circuit of an optional number of choppers are analyzed. The method is based on the application of Fourier analysis and the principle of superposition. An actual result is also presented.

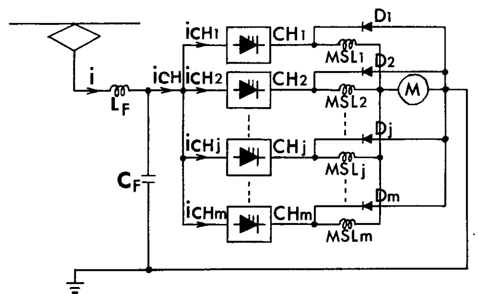
II. ANALYSIS

The calculation consists of Fourier analysis of each chopper current, obtaining the resultant current by superposing these and multiplying this resultant current by the filter transfer function in order to the content of each harmonic in the power line. In such calculation, the chopper is regarded as a constant current source.

Fig. 1 shows the multiple-phase chopper circuit consisting of m unit choppers. At the first time, we consider each chopper operates at a phase difference of $360/m$ degrees with a balanced conduction rate. In this circuit, the general Fourier representations of the current which flows in the j chopper can be summarized as



(a) Number of phases$=m$; number of multiple$=1$



(b) Number of phases$=m$; number of multiple$=m$

Fig. 1 Multiple-phase chopper circuits

$$i_{CHj}=\frac{a_{0j}}{2}+\sum_{n=1}^{\infty}\left(a_{nj}\cos\frac{2\pi n}{T}t+b_{nj}\sin\frac{2\pi n}{T}t\right) \quad \text{......(1)}$$

where a_{0j}, a_{nj}, and b_{nj} are the Fourier coefficients and T is the operating period of each chopper.

The compound current i_{CH} is given by the following:

$$i_{CH}=\sum_{j=1} i_{CHj}=\frac{A_0}{2}+\sum_{n=1}^{\infty}\left(A_n\cos\frac{2\pi n}{T}t+B_n\sin\frac{2\pi n}{T}t\right) \quad \text{..................(2)}$$

where

$$A_0=a_{01}+a_{02}+\ldots\ldots+a_{0j}+\ldots\ldots+a_{0m}$$
$$A_n=a_{n1}+a_{n2}+\ldots\ldots+a_{nj}+\ldots\ldots+a_{nm}$$
$$B_n=b_{n1}+b_{n2}+\ldots\ldots+b_{nj}+\ldots\ldots+b_{nm} \quad \text{...............(3)}$$

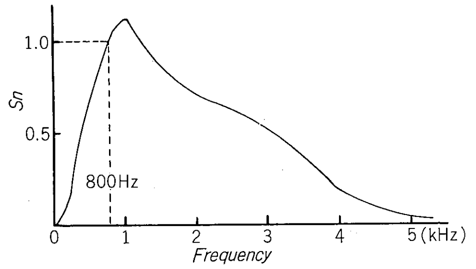
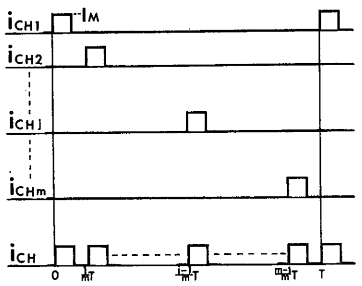
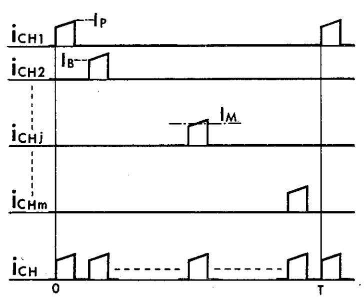


Fig. 2 **Telephone influence factor S_n (weighting function)**



(a) $\mu=0$



(b) $\mu\neq 0$

Fig. 3 **Chopper phase currents ($\alpha<1/m$)**

The rms. value I_{CHn} of the nth hamonic current of this compound current can be calculated by the following equation:

$$I_{CHn}=\sqrt{\frac{A_n{}^2+B_n{}^2}{2}}\ . \qquad \text{..................(4)}$$

Therefore, considering the frequency characteristics of the filter, the rms. value I_n of the nth harmonic current of the resultant current from the power line via the reactor in the LC input filter is given by the following:

$$I_n=\left|\frac{\dfrac{1}{jnwC_F}}{jnwL_F+\dfrac{1}{jnwC_F}}\right| I_{CHn}=\frac{1}{\left|1-\left(\dfrac{nf_{CH}}{f_F}\right)^2\right|}I_{CHn} \qquad \text{..............(5)}$$

where the chopper elementary frequency $f_{CH}=1/T$, $w=2\pi f_{CH}$, and the filter resonance frequency $f_F=1/2\pi\sqrt{L_FC_F}$.

The equivalent disturbance current J_p which serves as the means of evaluating the inductive interference in communication lines nearby the power line can be calculated by the following:

$$J_p=\sqrt{\sum_{n=1}^{\infty}(s_nIn)^2} \qquad \text{..................(6)}$$

where s_n is the telephone influence factor (see *Fig. 2*, 1953 CCITT specification).

If the Fourier coefficients $a_{n1}\sim a_{nm}$ and $b_{n1}\sim b_{nm}$ are obtained, all of the required values can be obtained using the preceding calculations. Now it becomes a serious problem to get the Fourier coefficients.

A. When Pulsating Factor is Zero

The current waveforms in this case are as shown in *Fig. 3 (a)*. These waveforms are for the case when the chopper operates as an ideal switch, the load side inductance is sufficiently large, and the load current is completely flat. Using the Fourier method, the coefficients can be obtained as shown in the following:

$$a_{n1}=\frac{I_M}{\pi n}\sin 2\pi n\alpha$$

$$a_{n2}=a_{n1}\cos 2\pi n\frac{1}{m}-b_{n1}\sin 2\pi n\frac{1}{m}$$

$$\vdots$$

$$a_{nj}=a_{n1}\cos 2\pi n\frac{j-1}{m}-b_{n1}\sin 2\pi n\frac{j-1}{m}$$

$$\vdots$$

$$a_{nm}=a_{n1}\cos 2\pi n\frac{m-1}{m}-b_{n1}\sin 2\pi n\frac{m-1}{m}$$

$$b_{n1}=\frac{I_M}{\pi n}(1-\cos 2\pi n\alpha)$$

$$b_{n2}=a_{n1}\sin 2\pi n\frac{1}{m}+b_{n1}\cos 2\pi n\frac{1}{m}$$

$$\vdots$$

$$b_{nj}=a_{n1}\sin 2\pi n\frac{j-1}{m}+b_{n1}\cos 2\pi n\frac{j-1}{m}$$

$$\vdots$$

$$b_{nm}=a_{n1}\sin 2\pi n\frac{m-1}{m}+b_{n1}\cos 2\pi n\frac{m-1}{m} \qquad \text{.........(7)}$$

where I_M is the load current per a phase (phase current) and α is the chopper conduction rate (balanced).

B. When Pulsating Factor is not Zero

The current waveforms in this case are shown in *Fig. 3 (b)*. The load current is almost linear in such cases. Therefore, the coefficients are as in the following:

$$a_{n1}=\left(\frac{\delta+\mu}{\pi n}\right)I_M\sin 2\pi n\alpha-\frac{\mu}{2(\pi n)^2\alpha}I_M(1-\cos 2\pi n\alpha)$$

$$b_{n1}=\left(\frac{\delta+\mu}{\pi n}\right)I_M(1-\cos 2\pi n\alpha)-\frac{\mu}{\pi n}I_M+\frac{\mu}{2(\pi n)^2\alpha}I_M\sin 2\pi n\alpha \qquad \text{..................(8)}$$

and all other coefficients are the same as in (7) where the pulsating factor $\mu=(I_p-I_B)/I_M$; $\delta=I_B/I_M=1-\mu/2\ (\leqq 1)$; I_M is the average load current ; I_p is the peak value of load current ; and I_B is the bottom value of load current.

From the two preceding analyses, it is evident that the harmonic current first becomes about $1/n$ according to (7) or (8) and then becomes $1/n^3$ at the filter of (5). Theoretically, the chopper conduction rate when the harmonic current is completely zero must fulfill the following :

Table 1 An aspect of harmonic currents

Harmonic currents	α value when I_n becomes zero
I_1	$\alpha=0,\ 1$
I_2	$\alpha=0,\ 1/2,\ 1$
I_3	$\alpha=0,\ 1/3,\ 2/3,\ 1$
⋮	⋮
I_n	$\alpha=0,\ 1/n,\ \ldots\ldots,\ 1$

$$2\pi n\alpha=2\pi l.$$

Therefore

$$\alpha=\frac{l}{n}(\leqq 1),\ l=0,\ 1,\ 2,\ 3,\ \ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

This case is shown in *Table 1*.

When some phases are absent, the coefficient a_x and b_x for these phases can be zero. An example of the calculation of numerical values is for a four-phase chopper. In this case, the chopper elementary frequency f_{CH} is 220 Hz (therefore, the resultant frequency is 880 Hz), the filter resonance frequency f_F is 42 Hz, and the pulsating factors of the motor current μ are 0 and 0.3 ($\delta=0.85$). The results are shown in *Figs. 4* and *5*. These figures show quantitatively that when there is no phase absent, only the $4K$ components of the elementary frequency component appear, but when some phase is absent, the K components appear. It is also evident that because of slight influence of μ we need not consider the pulsation on the load current seriously. Rather we should consider the phase absence seriously. *Table 2* shows typical numerical values from the results of calculations. The other components can

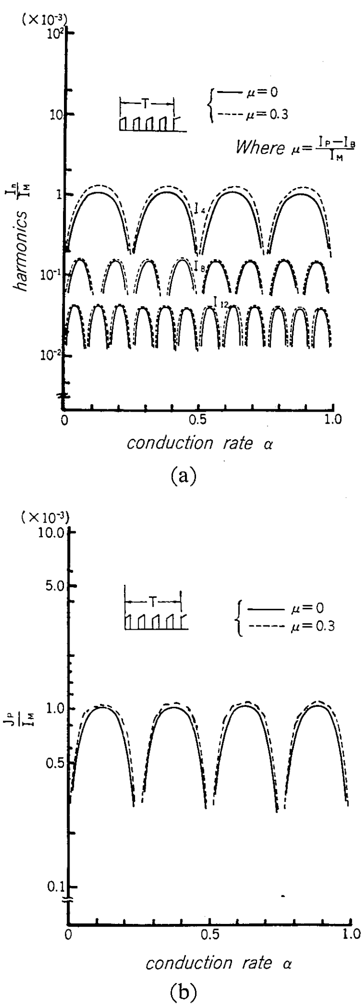


Fig. 4 Harmonic currents I_n and equivalent disturbance current J_p in four-phase chopper

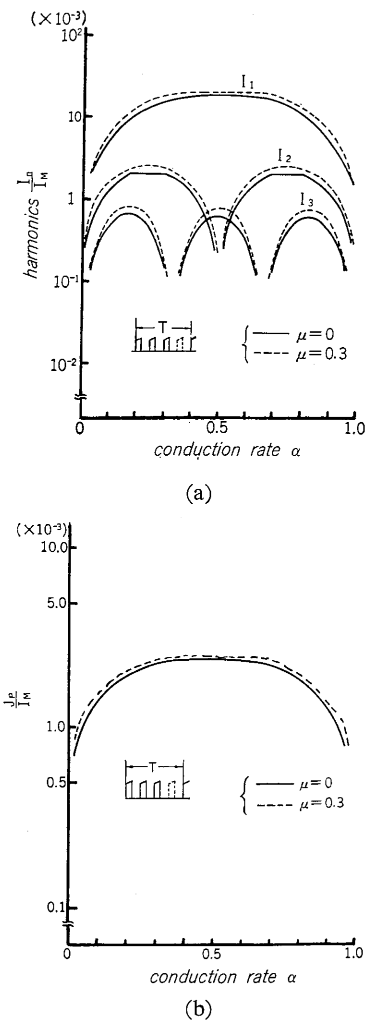
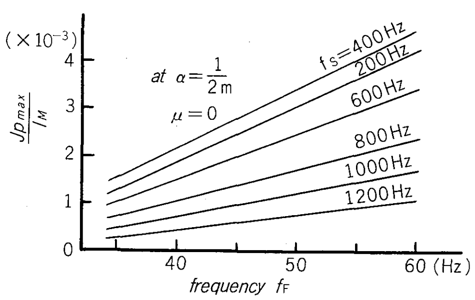


Fig. 5 I_n and J_p for one-phase absence in four-phase chopper

***Table 2* Typical calculated values of I_n and J_p in several cases**

Case			I_n	J_p
No phase absence (Fig. 4)		$\mu=0$	$I_4 \leqq 0.980$ A/1,000 A $I_8 \leqq 0.075$ A/1,000 A $I_{12} \leqq 0.022$ A/1,000 A $I_{16} \leqq 0.015$ A/1,000 A ⋮	$J_p \leqq 1.04$ A/1,000 A
		$\mu=0.3$	$I_4 \leqq 0.998$ A/1,000 A $I_8 \leqq 0.078$ A/1,000 A $I_{12} \leqq 0.024$ A/1,000 A $I_{16} \leqq 0.015$ A/1,000 A ⋮	$J_p \leqq 1.05$ A/1,000 A
1 phase absence (Fig. 5)		$\mu=0$	$I_1 \leqq 17.0$ A/1,000 A $I_2 \leqq 2.07$ A/1,000 A $I_3 \leqq 0.603$ A/1,000 A $I_4 \leqq 0.733$ A/1,000 A ⋮	$J_p \leqq 2.39$ A/1,000 A
		$\mu=0.3$	$I_1 \leqq 17.2$ A/1,000 A $I_2 \leqq 2.10$ A/1,000 A $I_3 \leqq 0.606$ A/1,000 A $I_4 \leqq 0.740$ A/1,000 A ⋮	$J_p \leqq 2.41$ A/1,000 A
2 phase absence	CH_2, CH_4 absence (correspond to a 2-phase chopper)	$\mu=0$	$I_2 \leqq 4.14$ A/1,000 A $I_4 \leqq 0.488$ A/1,000 A $I_6 \leqq 0.145$ A/1,000 A $I_8 \leqq 0.061$ A/1,000 A ⋮	$J_p \leqq 2.32$ A/1,000 A
		$\mu=0.3$	$I_2 \leqq 4.16$ A/1,000 A $I_4 \leqq 0.494$ A/1,000 A $I_6 \leqq 0.146$ A/1,000 A $I_8 \leqq 0.061$ A/1,000 A ⋮	$J_p \leqq 2.34$ A/1,000 A
	CH_2, CH_3 absence	$\mu=0$	$I_1 \leqq 24.1$ A/1,000 A $I_3 \leqq 0.852$ A/1,000 A $I_4 \leqq 0.488$ A/1,000 A $I_5 \leqq 0.186$ A/1,000 A ⋮	$J_p \leqq 3.28$ A/1,000 A
		$\mu=0.3$	$I_1 \leqq 24.2$ A/1,000 A $I_3 \leqq 0.860$ A/1,000 A $I_4 \leqq 0.493$ A/1,000 A $I_5 \leqq 0.187$ A/1,000 A ⋮	$J_p \leqq 3.29$ A/1,000 A
Note			$m=4$ $f_{CH}=220$ Hz $f_F=42$ Hz	



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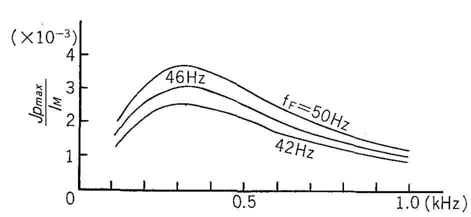
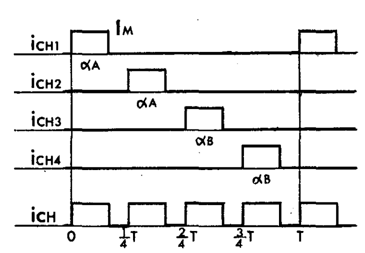


(b)

Fig. 6 **Relations of $J_{p\,max}$, f_F, and f_s**
f_F: filter resonance frequency
f_s: resultant frequency ($f_s=mf_{CH}$)



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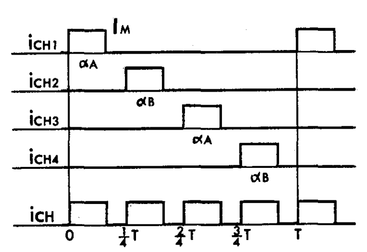


(b)

Fig. 7 **Chopper phase currents with unbalanced conduction rate in four-phase chopper ($\alpha < 1/4$)**

be calculated approximately by multiplying this elementary frequency component by the $1/n^3$ according to the preceding consideration.

Fig. 6 shows the relation between the filter resonance frequency f_F and the resultant frequency of the chopper f_s ($f_s = mf_{CH}$). This is for the case when the chopper conduction rate is $1/2m$ at which the harmonic current and the equivalent disturbance current are maximum.

At the next time, we consider choppers with unbalanced conduction rate. The analysis is the same as the aforementioned. The case is for a four-phase chopper. The current is completely flat, and the chopper conduction rate is unbalanced as shown in *Fig. 7*.

C. When Conduction Rate is α_A, α_A, α_B, α_B

At this time the coefficients $a_{n1} \sim a_{n4}$ and $b_{n1} \sim b_{n4}$ can be given by the following:

$$a_{n1} = \frac{I_M}{\pi n} \sin 2\pi n \alpha_A$$

$$a_{n2} = a_{n1} \cos 2\pi n \frac{1}{4} - b_{n1} \sin 2\pi n \frac{1}{4}$$

$$a_{n3} = \frac{I_M}{\pi n} \left(\sin 2\pi n \frac{1}{2} \cos 2\pi n \alpha_B + \cos 2\pi n \frac{1}{2} \sin 2\pi n \alpha_B - \sin 2\pi n \frac{1}{2} \right)$$

$$a_{n4} = a_{n3} \cos 2\pi n \frac{1}{4} - b_{n3} \sin 2\pi n \frac{1}{4}$$

$$b_{n1} = \frac{I_M}{\pi n} (1 - \cos 2\pi n \alpha_A)$$

$$b_{n2} = a_{n1} \sin 2\pi n \frac{1}{4} + b_{n1} \cos 2\pi n \frac{1}{4}$$

$$b_{n3} = \frac{I_M}{\pi n} \left(-\cos 2\pi n \frac{1}{2} \cos 2\pi n \alpha_B + \sin 2\pi n \frac{1}{2} \sin 2\pi n \alpha_B + \cos 2\pi n \frac{1}{2} \right)$$

$$b_{n4} = a_{n3} \sin 2\pi n \frac{1}{4} + b_{n3} \cos 2\pi n \frac{1}{4}. \quad \ldots\ldots\ldots\ldots\ldots (10)$$

D. When Conduction Rate is α_A, α_B, α_A, α_B

At this time the coefficients $a_{n1} \sim a_{n4}$, ane $b_{n1} \sim b_{n4}$ can be expressed by the following:

$$a_{n1} = \frac{I_M}{\pi n} \sin 2\pi n \alpha_A$$

$$a_{n2} = \frac{I_M}{\pi n} \left(\sin 2\pi n \frac{1}{4} \cos 2\pi n \alpha_B + \cos 2\pi n \frac{1}{4} \sin 2\pi n \alpha_B - \sin 2\pi n \frac{1}{4} \right)$$

$$a_{n3} = a_{n1} \cos 2\pi n \frac{1}{2} - b_{n1} \sin 2\pi n \frac{1}{2}$$

$$a_{n4} = a_{n2} \cos 2\pi n \frac{1}{2} - b_{n2} \sin 2\pi n \frac{1}{2}$$

$$b_{n1} = \frac{I_M}{\pi n} (1 - \cos 2\pi n \alpha_A)$$

$$b_{n2} = \frac{I_M}{\pi n} \left(-\cos 2\pi n \frac{1}{4} \cos 2\pi n \alpha_B + \sin 2\pi n \frac{1}{4} \sin 2\pi n \alpha_B + \cos 2\pi n \frac{1}{4} \right)$$

$$b_{n3} = a_{n1} \sin 2\pi n \frac{1}{2} + b_{n1} \cos 2\pi n \frac{1}{2}$$

$$b_{n4} = a_{n2} \sin 2\pi n \frac{1}{2} + b_{n2} \cos 2\pi n \frac{1}{2}. \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Examples of the numerical calculation for these cases are shown in *Fig. 8* and *9*. In these cases, the chopper elementary frequency f_{CH} is 220 Hz, the filter resonance frequency f_F is 42 Hz, and the conduction rate unbalance is 10 percent.

When the chopper conduction rate is unbalanced, there is no mutual cancellation of the phases, and even when no phase is absent, it is evident that K component of the chopper elementary frequency component appears.

E. Other Cases

Other cases, for example when all of the chopper conduction rate are unbalanced and when the load

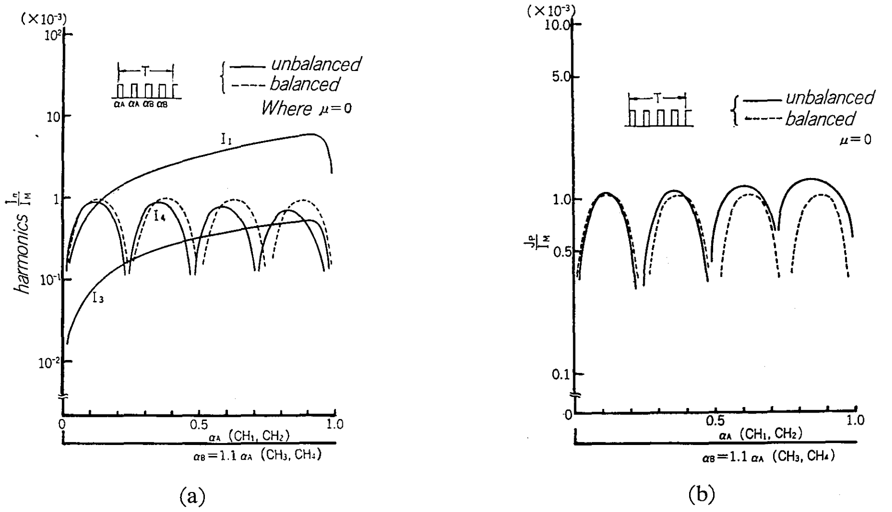
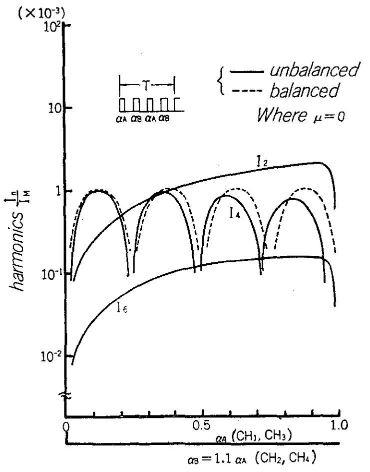


Fig. 8 I_n and J_p in four-phase chopper with α_A, α_A, α_B, α_B conduction rate in *Fig. 7 (a)*



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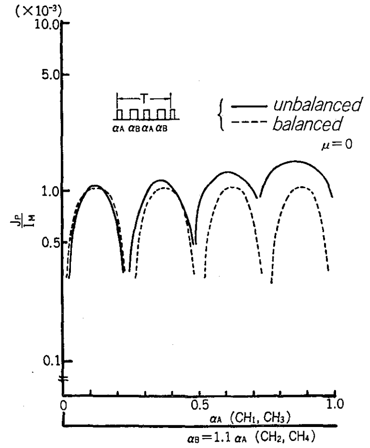
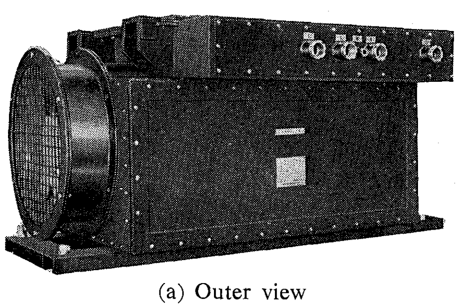
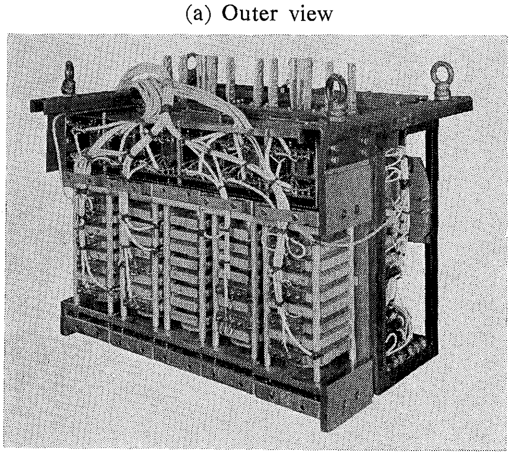


(b)

Fig. 9 I_n and J_p in four-phase chopper with α_A, α_B, α_A, α_B conduction rate in *Fig. 7 (b)*



(a) Outer view



(b) Chopper part

Fig. 10 Oil immersed chopper equpiment

currents of each chopper are unbalanced, also can be calculated. After thorough investigations of the results of the analyses under the conditions given previously, it is possible to make the chopper control equipment as simple as required, to decrease the number of chopper phases to the required level, to make the filter as small as necessary, and to avoid any increases in price or drops in reliability.

III. TEST RESULTS

Fig. 10 shows the thyristor chopper equipment manufactured by Fuji Electric Company, Ltd., and *Table 3* shows the principal items of this chopper.

Fig. 11 shows the measured results of the harmonic currents, and *Table 4* lists the measured results of J_p. These values are the maximum values during test running. In this test sufficient results were obtained and it appeared that our current countermeasures were sufficient.

IV. CONCLUSION

This paper has described a general method for obtaining the harmonic current and the equivalent disturbance current which occur in electric rolling stock controlled by several choppers.

***Table 3* Principal electrical rating of DC chopper in test run**

Composition	M+M′
Main motor capacity	85 kW (375 V × 227 A) × 4 unit
Car line voltage	750 V (450 V～825 V)
Maximum acceleration current	486 A/motor (α=3.5)
Maximum brake current	325 A/motor (β=3.5)
Motor connection	2S•2P permanent
Chopper connection	2-phase double
Chopper frequency	200 Hz (total 400 Hz)
Main motor current pulsation factor	10%
Control system	Average current value control, set frequency phase angle control system
Chopper circuit system	Special double reversing system
Cooling system	Oil immersed air cooling (ambient temperature −30°C ～+40°C)
Commutating capacitor capacity	165 μF
Smooth reactor inductance	5 mH
Filter reactor inductance	7 mH
Filter capacitor capacity	1,800 μF
Filter resonance frequency	45 Hz

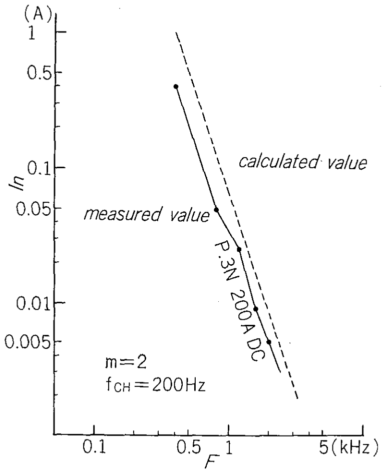


Fig. 11 Test results

Table 4 J_p **of power source current in test run**

Speed	Supply current (A)	J_p (A)
Powering 1 notch	30	0.005
Powering 2 notch	400	0.045
Powering 3 notch	400	0.055
Braking 1 notch	100	0.045
Braking 2 notch	140	0.034
Braking 3 notch	180	0.05
Braking 4 notch	200	0.05
Braking 5 notch	230	0.06
Braking 6 notch	230	0.063
Braking 7 notch	260	0.073
Powering 2 notch (chopper single phase)	200	0.05

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