ARTIFICIAL ARC-BACK TEST AT THE KAMBARA WORKS OF THE NIPPON LIGHT METAL CO.

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I. INTRODUCTION

In reconstructing the Niigata Works of the Nippon Light Metal Co., it was planned to increase the current capacity of the aluminium pot line from 40 kA to 80 kA by rearranging the parallel operation of the large number of mercury arc rectifiers of our makes that have been in service for years. Being the first and largest current aluminium pot line ever contemplated in Japan, duties of the rectifier equipment against fault current are enlarged and a countermeasure is required to be set up. Undoubtedly, the calculation of arc-back currents is one of the most difficult problems encountered in the application of the rectifiers and the determination of the proper current value is also necessary for the determination of the maximum duties of the rectifier, rectifier transformer and circuit breakers on the a–c and d–c sides. But arc-back is one of the phenomena of probability and its measurements is very difficult and left unclarified. In America, when a large current aluminium pot line was built, an artificial arc-back test was first made to take necessary measurements. It was the Nippon Light Metal Co. that made the first artificial arc-back test in Japan the test being conducted at the Kambara Works in December, 1956. Through these tests, the method of calculation has been established and the maximum duty of the rectifier equipment was made clear.

When an arc-back occurs, the rectifier transformer is short-circuited on the secondary side and simultaneously reverse-current flows from the d–c bus. The former is called a–c feed current and the latter d–c feed current. These fault currents are nearly always higher than currents resulting from the so called two or three phase short-circuit on the rectifier transformer. Various difficulties were encountered in the calculation of arc-back currents and some of the most principal ones are given below.

The rectifier transformer is usually of six phase double-three phase star connection with a interphase reactors and the presence of the impedance causes a big difference in results depending on whether it is inserted on the primary side or secondary side.

Again the effect of the impedance of a–c source system is complex and fairly great. The performance of the interphase reactor is also complex showing one of the nonlinear characteristic features as the case with d–c reactors which are used most commonly. Arc-drop in the rectifier, which is also one of the nonlinear characteristic features, is usually treated as an equivalent resistance characteristic. Likewise, an equivalent d–c supply source is determined according to the nature of a supply source. The phase condition at the occurence of arc-back has an important bearing on the arc-back currents. Therefore, load condition and phase control factor become a matter of importance. It is generally believed that in a majority of cases arc-back occurs concurrently with the end of commutation.

II. TEST RESULTS

The installation at Kambara Works includes 26 water cooled, multi-anode steel tank rectifiers of rating 750 V, 6,000 A and ten of them were used for the test. The rectifier transformers were of three-phase, delta connection for the primary side and twelve-phase, quadruple, zig-zag star connection for the secondary side, being rated at 11/0.74 kV,

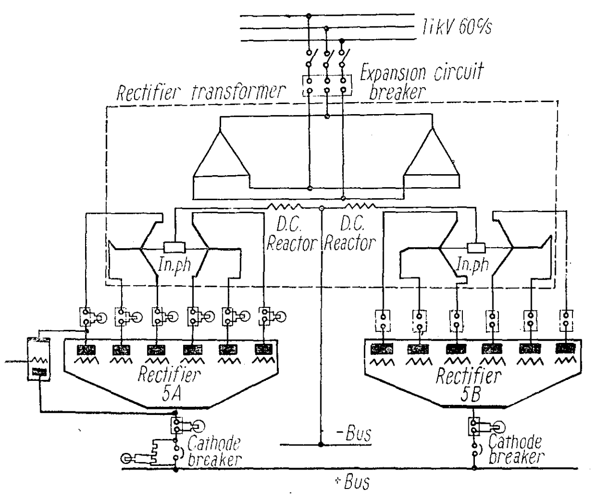

Fig. 1. Testing circuits of artificial arc-back

10,600 kVA and in a combination of two rectifiers for each transformer as shown in Fig. 1.

The receiving power transformers were of three-phase, 60 cycle, 77/11 kV, 35,000 kVA and two banks of them were used in the test. The rectifiers, 5 A and 8 A units, on which arc-back was premeditated, were synchronized concurrently with the end of commutation at the anode u, and a single anode rectifier was reversely connected to it and current is passed to produce artificial arc-back currents in the rectifiers as planned. Thus, voltage and current measurements were taken on the a–c and d–c sides of it. Test was conducted in this manner and in accordance with the schedule as given in Table 1, and the effect of the impedance of the source system, the number of rectifiers connected in parallel operation and position of the rectifier in which arc-back occured was experimentally studied.

Table 1. Test schedule

Test number	Number of banks step down transf.	Number of parallel running rectifier set	Number of arc-back rectifier
T–3	1	8	5A
T–4	2	8	5A
T–5	1	10	5A
T–6	2	10	5A
T–7	1	10	8A
T–8	2	10	8A

T3~4: 8 sets of No. 5, 7, 8 and 11
T5~8: 10 sets of No. 5, 7, 8, 10 and 11

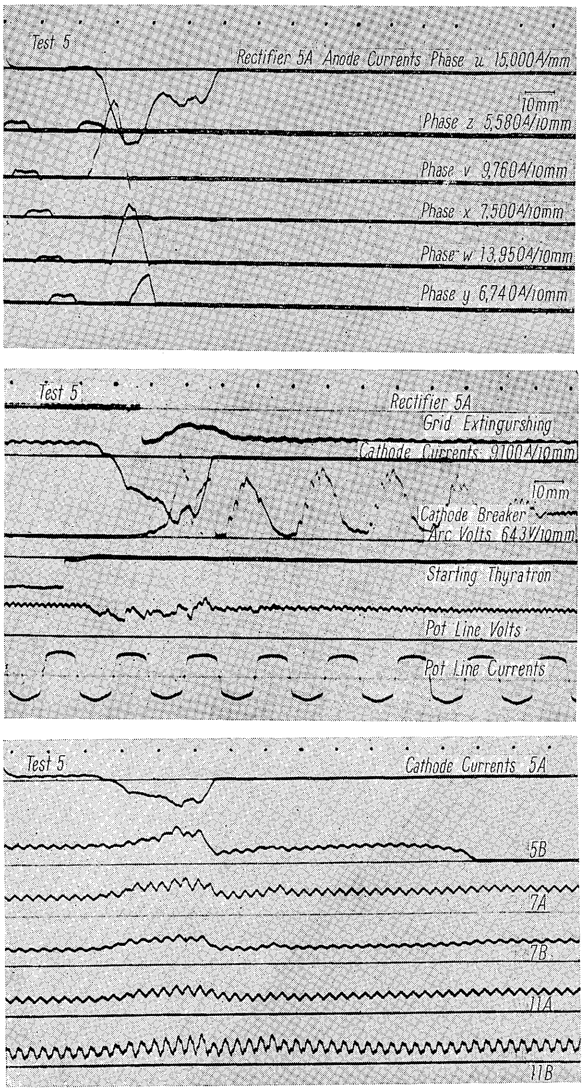

Fig. 2. Typical oscillogram of arc-back with arc suppression, test T–5

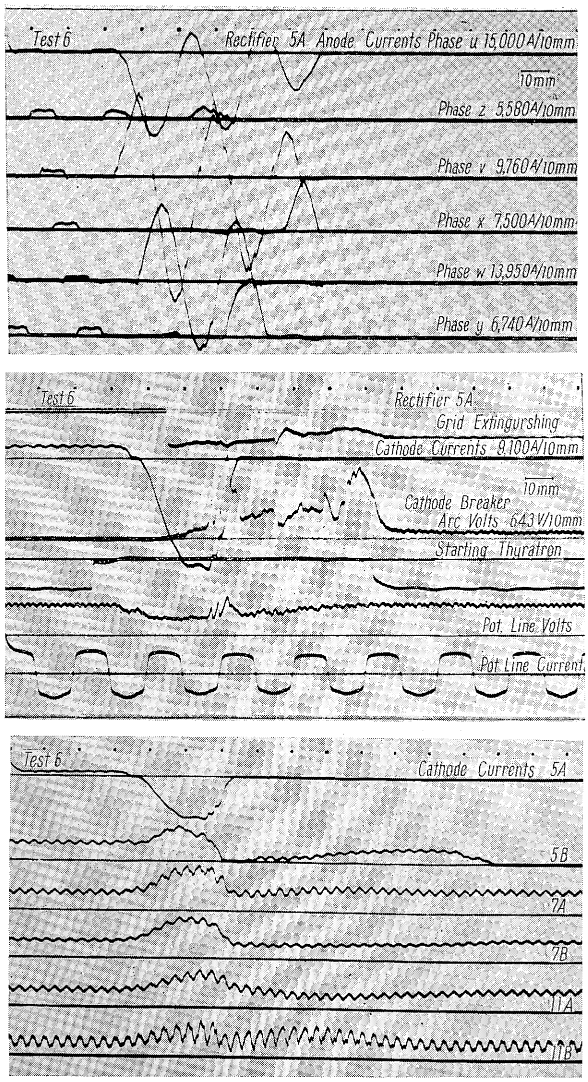

Fig. 3. Typical oscillogram of pessimistic arc-back, test T–6

D–c voltage and current on the initial condition of artificial arc-back were 650 V and 36,000 A respectively and phase control factor 97%. Representative test oscillograms are shown in Fig. 2, 3 and 4 and the maximum values of fault currents in Table 2.

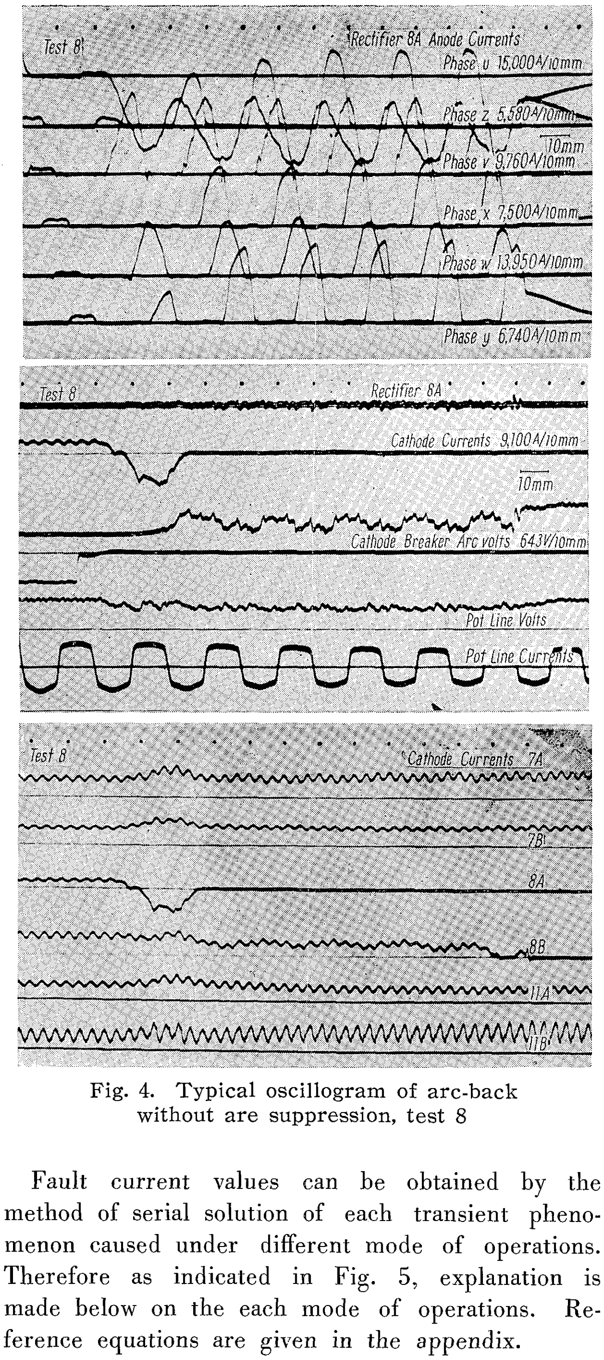

Fig. 4. Typical oscillogram of arc-back without are suppression, test 8

Fault current values can be obtained by the method of serial solution of each transient phenomenon caused under different mode of operations. Therefore as indicated in Fig. 5, explanation is made below on the each mode of operations. Reference equations are given in the appendix.

Table 2. Fault currents in amperes

Test number	Crest value in faulty anode (phase u)	Crest value in normal anode (phase v)	Crest value with a–c feed only		Crest value with d–c feed only
			in faulty anode	in normal anode	
T–3	39,300	26,900	38,120	26,900	23,200
T–4	44,400	30,300	40,940	30,900	20,800
T–5	39,800	27,000	36,500	28,060	21,000
T–6	45,000	29,300	42,280	29,300	36,000
T–7	38,200	27,200	35,200	27,200	10,000
T–8	40,000	28,800	37,440	30,300	10,900

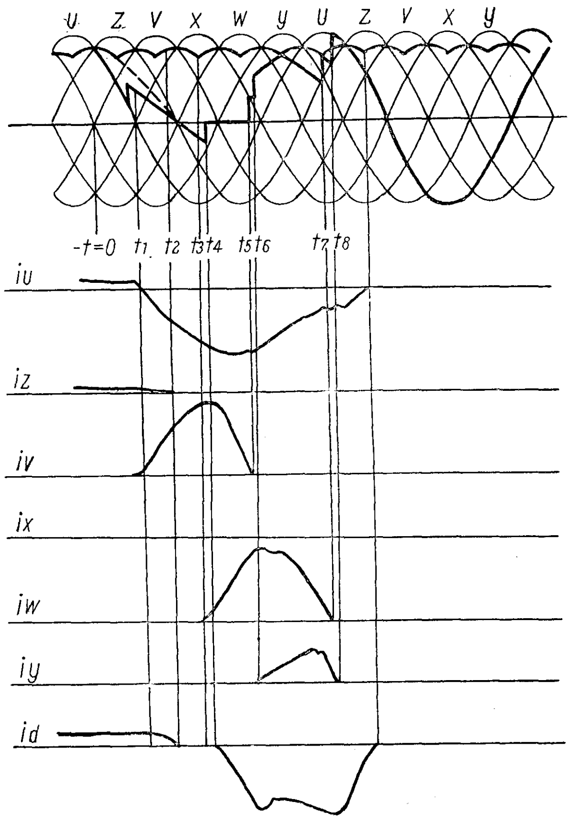

Fig. 5. Fault currents duning arc-back with a–c feed and d–c feed in a 6 phase double star rectifier

Interval I—t_1 to t_2

Assumed that arc-back has occured at time point t_1—that is— it has lagged behind the intersection point of voltage phases u and v by delay angle and overlapping angle. Then each of the phase u, v and z maintains its normal current flow and no change is observed in the performance of the interphase reactor. Under this condition, the sum of u and v phase currents is equal to that of phase z and the total sum of these three phase currents is equal to the d–c current. That is, phase z current is the same as d–c current in waveform decreased by one half in value.

Interval II—t_2 to t_3

Between time point t_2 at which excitation current of the interphase reactor becomes equal to one half of d–c current in value and time point t_4 at which the interphase reactor has its total linkage of magnetic flux reversed—the interphase reactor suddenly increases its reactance to such an extent that it absorbs, with the d–c reactor, a greater percentage of the d–c feed voltage generated by the normal rectifiers in parallel operation and lowers the potential at neutral point of the normal secondary star of the rectifier transformer on which arc-back is premeditated. Then phase x does not ignite at its ignition time point and d–c feed current commences its reverse flow lagging behind time point t_4. Therefore, if voltage e is impressed on the interphase reactor and Φ is total linkage of flux, the following equation exists

$$\Phi=\int_{t_2}^{t_4} e \ dt.$$

Interval III—t_3 to t_4

If phase w ignites at time point t_3 with d–c feed current practically zero, the so-called three-phase short-circuit a–c feed current occurs and the faulty secondary star of the rectifier transformer is completely short-circuited.

Interval IV—t_4 to t_5

If d–c feed current is generated at time point t_4, a–c feed voltage becomes zero on account of three-phase short-circuit of the rectifier transformer and the current that flows is a resultant of the d–c voltage short-circuited by the reactor and resistance. But because of the iron core of the interphase reactor and d–c reactor, its reactance becomes gradually saturated. As a result, its reactance has a tendency of becoming constant.

Interval V—t_5 to t_6

Arc extinction of the phase v takes place at time point t_5.

Interval VI—t_6 to t_7

At time point t_6, the interphase reactor and d–c reactor become saturated rendering phase y to ignite, a voltage equal in value is induced in each phase winding of the interphase reactor and the d–c feed current shows a tendency of decline in its upward trend.

Interval VII—t_7 to t_8

At time point t_7 arc extinction takes place of the phase w.

Interval VIII—t_8 to —

If arc extinction of the phase y takes place at time point t_8 and the grid extinguishing holds on, then, no current flow is allowed but completely blocked except by phase u. Therefore, the single-phase a–c feed voltage of phase u and series voltage of the d–c feed voltage are short-circuited. In this section, d–c feed current is totally interrupted by a d–c high-speed circuit breaker. Accordingly, considering the characteristic of a d–c high-speed circuit breaker, the time point of interruption can be determined.

III. STUDY ON TEST RESULTS

Fault current:

Fault current can be computed with fair accuracy by the method given in the appendix. In the most simplified method of computation of a–c feed current, the first maximum value of a–c feed current is obtained in the following manner; the circuit constant $\frac{R_C}{X_C}$ is made a variable; and making the phase control factor as a parameter, multiples of the ordinary single-phase a–c short-circuit current $\frac{\sqrt{2}E_S}{Z_C}$ are fixed before hand, neglecting initial value. In Table 3 is shown the comparison between the calculated and actual values.

Table 3. Fault currents with a-c feed only in amperes

Number of banks step down transf.	Test		Calculation	
	Normal anode	Faulty anode	Normal anode	Faulty anode
1	28,060	38,120	30,800	39,600
2	30,900	42,280	34,300	44,000

The effect that the impedance of the receiving power transformer on the fault current is 15∼20% in excess with two tanks than one tank, while the number of rectifiers and the location of arc-back rectifier cause practically no effect on the fault current. Because of a great number of the sound rectifiers in parallel operation the number of rectifiers, in general, whether be it eight or ten in number, does make little difference in effect due to the predominance of the impedance of the arc-back rectifier circuit. The effect of the arc-back rectifier due to its location in mainly attributable to the impedance of the bus bar and is negligible because the impedance of the bus bars itself is negligibly small.

The maximum value of the d–c feed current is reached when an arc-back is caused on each phase

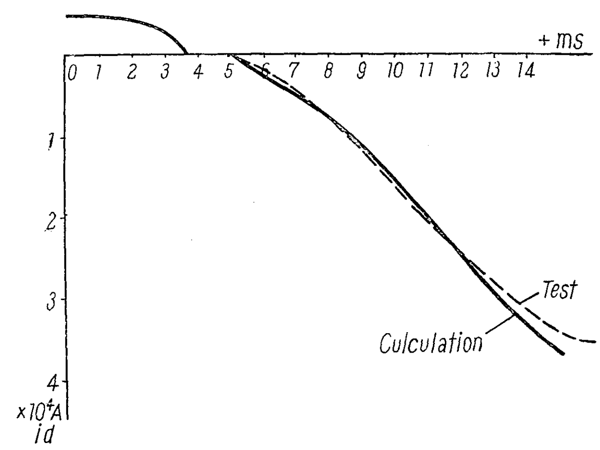

Fig. 6. Wave form during arc-back with d–c feed only

in succession as shown in Fig. 3 and its magnitude and the maximum value of rate of rise in current can be obtained by the same caluclation method as applied in the mode of Interval IV operation.

Approximate equations dealing with these values are given below in which E_d stands for d–c feed voltage, L_3 and R_3 for reactance and resistance of the circuit respectively.

$$id \doteqdot \frac{E_d}{R_3}\left[1-\epsilon^{-\frac{R_3}{L_3}t}\right] \quad \ldots\ldots\ldots\ldots\ldots 2)$$

$$\frac{di_d}{dt} \doteqdot \frac{E_d}{L_3} \quad \ldots\ldots\ldots\ldots\ldots 3)$$

In Fig. 6 is shown an comparison of the calculated and actual values.

Duty of d–c high-speed circuit breaker :

The maximum value of current can be obtained by the formular mentioned above and its corresponding recovering voltage is comparatively relieved as shown in Fig. 4 if the arc-back rectifier is not in grid extinguishing. But, if grid-extinguishing as shown in Fig. 2, recovering voltage—a series voltage resulted by the aggregation of the d–c feed voltage and single phase a–c voltage of the arc-back phase u—as explained in connection with the Interval VIII, is divided into two elements—two impedance regained in proportion to the recovery of insulation strength of the d–c circuit breaker and rectifier after the arc-back current has subsided. In the test because of the single-anode rectifier connected inversely and the potentiometer connected for measurement purpose, the recovering voltage was not measurred. This fact has presented one of the important problems in determing the recovering voltage of a d–c circuit-breaker. Of the voltage drop, due to the impedance of a supply source system, the one resulted from short-circuit of one rectifier transformer is determinant, when a d–c feed current starts its reverse flow at the time point but is magnified in proportion to the overloading of the normal rectifiers in parallel. The final d–c feed current value must be computed with due cosideration of the voltage drop caused by short-circuit of the entire rectifier equipments. To protect against any d–c short-circuit overload current, the normal rectifiers in parallel operation will be grid extinguished.

Control and protection method :

In general, the inductance of an aluminum pot line is fairly large and its time constant is also large. Therefore, the effect of a feed back current flow from the aluminum pot line at its initial stage is practically negligible then a selective circuit breaking method to make constant feeding by clearing out an arc-back rectifier of a group of rectifiers in parallel with a d–c high-speed circuit-breaker has been attested. Similarly, the interphase reactor which functions as one of the protective reactor through its delaying action against d–c feed current and the d–c high-speed circuit-breaker which acts as a release at small positive current perform the reduction of the current duty of the d–c circuit-breaker considerably. At present, as one of the protective methods, grid extinguishing is adopted in practice through the use of a d–c over-load relay. But if grid extinguishing is made through the use of a thyratron relay which operates on the arc-back phase current and if phase w is successfully grid extinguished then the current duty of the rectifier transformer will be greatly relieved.

Appendix

The symbols which appears in the equations are defined as follows.

$\omega = 2\pi f$ = angular frequency. t = time in seconds.

E_d = average voltage of d–c source during feed back in volts.

E_s = transformer secondary winding line to neutral voltage in rms volts.

i_d = instantaneous direct current in amperes.

i_u = instantaneous reverse u phase current in amperes.

i_v = instantaneous forward v phase current in amperes.

i_w = instantaneous forward w phase current in amperes.

The second subscript in i_d, i_u, i_v, i_w and so on indicates initial currents at the beginning of the respective intervals.

L_d = total inductance in henrys, of d–c feed back circuits.

L_p = inductance of inter phase reactor in henrys per phase.

l_p = leakage inductance of " "

L_c = commutating inductance in henrys per phase.

R_d = total resistance in ohms, of d–c feed back circuits.

R_p = resistance of interphase reactor in ohms per phase.

R_c = commutating resistance in ohms per phase.

Z_c = commutating impedance in ohms per phase, $\tan\phi_c = \omega L_c / R_c$

Interval I—t_1 *to* t_2

$$1)\quad i_u=\frac{\sqrt{6}E_s}{2Z_c}\left[\sin\left(\omega t-\frac{\pi}{6}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_1)}\sin\left(\omega t_1-\frac{\pi}{6}-\phi_c\right)\right]+\left(i_{u1}-\frac{i_{d1}}{4}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_1)}+\frac{i_d}{4}$$

$$2)\quad i_v=\frac{\sqrt{6}E_s}{2Z_c}\left[\sin\left(\omega t-\frac{\pi}{6}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_1)}\sin\left(\omega t_1-\frac{\pi}{6}-\phi_c\right)\right]+\left(i_{v1}-\frac{i_{d1}}{4}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_1)}-\frac{i_d}{4}$$

$$3)\quad i_d=2i_z=\frac{E_d}{R_1}\left[1-\varepsilon^{-\frac{R_1}{L_1}(t-t_1)}\right]+\frac{3}{4}\ \frac{\sqrt{2}E_s}{Z_1}\left[\sin\left(\omega t-\frac{2}{3}\pi-\phi_1\right)-\varepsilon^{-\frac{R_1}{L_1}(t-t_1)}\sin\left(\omega t_1-\frac{2}{3}\pi-\phi_1\right)\right]+i_{d1}\,\varepsilon^{-\frac{R_1}{L_1}(t-t_1)}$$

Interval II—t_2 *to* t_3

$$4)\quad i_u=\frac{\sqrt{6}E_s}{2Z_c}\left[\sin\left(\omega t-\frac{\pi}{6}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_2)}\sin\left(\omega t_2-\frac{\pi}{6}-\phi_c\right)\right]+i_{u2}\,\varepsilon^{-\frac{R_c}{L_c}(t-t_2)}$$

$$5)\quad i_v=\frac{\sqrt{6}E_s}{2Z_c}\left[\sin\left(\omega t-\frac{\pi}{6}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_2)}\sin\left(\omega t_2-\frac{\pi}{6}-\phi_c\right)\right]+i_{v2}\,\varepsilon^{-\frac{R_c}{L_c}(t-t_2)}$$

Interval III—t_3 *to* t_4

$$6)\quad i_u=\frac{\sqrt{2}E_s}{Z_c}\left[\sin\left(\omega t-\frac{\pi}{3}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}\sin\left(\omega t_3-\frac{\pi}{3}-\phi_c\right)\right]+i_{u3}\,\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}$$

$$7)\quad i_v=\frac{\sqrt{2}E_s}{Z_c}\left[\sin(\omega t-\phi_c)-\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}\sin(\omega t_3-\phi_c)\right]+i_{v3}\,\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}$$

$$8)\quad i_w=\frac{\sqrt{2}E_s}{Z_c}\left[\sin\left(\omega t-\frac{2}{3}\pi-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}\sin\left(\omega t_3-\frac{2}{3}\pi-\phi_c\right)\right]+i_{w3}\,\varepsilon^{-\frac{R_c}{L_c}(t-t_3)}$$

Interval IV—t_4 *to* t_5

$$9)\quad \Phi=\int_{t_2}^{t_4}e\,dt$$

$$10)\quad i_u=\frac{\sqrt{2}E_s}{Z_c}\left[\sin\left(\omega t-\frac{\pi}{3}-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}\sin\left(\omega t_4-\frac{\pi}{3}-\phi_c\right)\right]+\left(i_{u4}-\frac{i_{d4}}{3}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}+\frac{i_d}{3}$$

$$11)\quad i_v=\frac{\sqrt{2}E_s}{Z_c}\left[\sin(\omega t-\phi_c)-\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}\sin(\omega t_4-\phi_c)\right]+\left(i_{v4}+\frac{i_{d4}}{3}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}-\frac{i_d}{3}$$

$$12)\quad i_w=\frac{\sqrt{2}E_s}{Z_c}\left[\sin\left(\omega t-\frac{2}{3}\pi-\phi_c\right)-\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}\sin\left(\omega t_4-\frac{2}{3}\pi-\phi_c\right)\right]+\left(i_{w4}+\frac{i_{d4}}{3}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_4)}-\frac{i_d}{3}$$

$$13)\quad i_d=\frac{E_d}{R_3}\left[1-\varepsilon^{-\frac{R_3}{L_3}(t-t_4)}\right]+i_{d4}\,\varepsilon^{-\frac{R_3}{L_3}(t-t_4)}$$

Interval V—t_5 *to* t_6

$$14)\quad i_u=\frac{\sqrt{6}E_s}{2Z_c}\left[\varepsilon^{-\frac{R_c}{L_c}(t-t_5)}\cos(\omega t_5-\phi_c)-\cos(\omega t-\phi_c)\right]+\left(i_{u5}-\frac{i_{d5}}{2}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_5)}+\frac{i_d}{2}$$

$$15)\quad i_w=\frac{\sqrt{6}E_s}{2Z_c}\left[\varepsilon^{-\frac{R_c}{L_c}(t-t_5)}\cos(\omega t_5-\phi_c)-\cos(\omega t-\phi_c)\right]+\left(i_{w5}+\frac{i_{d5}}{2}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_5)}-\frac{i_d}{2}$$

$$16)\quad i_d=\frac{E_d}{R_2}\left[1-\varepsilon^{-\frac{R_2}{L_2}(t-t_5)}\right]+\frac{\sqrt{2}E_s}{2Z_2}\left[\sin(\omega t-\phi_2)-\varepsilon^{-\frac{R_2}{L_2}(t-t_5)}\sin(\omega t_5-\phi_2)\right]+i_{d5}\,\varepsilon^{-\frac{R_2}{L_2}(t-t_5)}$$

Interval VI—t_6 *to* t_7

$$17)\quad i_u=\frac{\sqrt{6}E_s}{2Z_c}\left[\varepsilon^{-\frac{R_c}{L_c}(t-t_6)}\cos(\omega t_6-\phi_c)-\cos(\omega t-\phi_c)\right]+\left(i_{u6}-\frac{i_{d6}+i_{y6}}{2}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_6)}+\frac{i_d+i_y}{2}$$

$$18)\quad i_w=\frac{\sqrt{6}E_s}{2Z_c}\left[\varepsilon^{-\frac{R_c}{L_c}(t-t_6)}\cos(\omega t_6-\phi_c)-\cos(\omega t-\phi_c)\right]+\left(i_{w6}+\frac{i_{d6}+i_{y6}}{2}\right)\varepsilon^{-\frac{R_c}{L_c}(t-t_6)}-\frac{i_d+i_y}{2}$$

$$19)\quad i_d\fallingdotseq i_y=\frac{R_d\,i_{d6}-E_d}{R_4}\left[1-\varepsilon^{-\frac{R_4}{L_4}(t-t_6)}\right]-\frac{\sqrt{2}E_s}{Z_4}\left[\sin(\omega t-\phi_4)-\varepsilon^{-\frac{R_4}{L_4}(t-t_6)}\sin(\omega t_6-\phi_4)\right]+i_{y6}\,\varepsilon^{-\frac{R_4}{L_4}(t-t_6)}$$

Interval VII—t_7 *to* t_8

$$20)\quad i_u=\frac{\sqrt{2}E_s}{2Z_5}\left[\sin\left(\omega t-\frac{2}{3}\pi-\phi_5\right)-\varepsilon^{-\frac{R_5}{L_5}(t-t_7)}\sin\left(\omega t_7-\frac{2}{3}\pi-\phi_5\right)\right]+\left(i_{u7}+\frac{i_{d7}}{2}\right)\varepsilon^{-\frac{R_5}{L_5}(t-t_7)}+\frac{i_d}{2}$$

$$21)\quad i_y=\frac{\sqrt{2}E_s}{2Z_5}\left[\sin\left(\omega t-\frac{2}{3}\pi-\phi_s\right)\pi-\varepsilon^{-\frac{R_5}{L_5}(t-t_7)}\sin\left(\omega t_7-\frac{2}{3}\pi-\phi_5\right)\right]+\left(i_{y7}+\frac{i_{d7}}{2}\right)\varepsilon^{-\frac{R_5}{L_5}(t-t_7)}-\frac{i_d}{2}$$

$$22)\quad i_d=\frac{E_d}{R_6}\left[1-\varepsilon^{-\frac{R_6}{L_6}(t-t_7)}\right]+\frac{\sqrt{6}E_s}{2Z_6}\left[\sin\left(\omega t-\frac{\pi}{6}-\phi_6\right)-\varepsilon^{-\frac{R_6}{L_6}(t-t_7)}\sin\left(\omega t_7-\frac{\pi}{6}-\phi_6\right)\right]+i_{d7}\,\varepsilon^{-\frac{R_6}{L_6}(t-t_7)}$$

Interval VIII—t_8 *to* –

$$23)\quad i_u=i_d=\frac{E_d}{R_7}\left[1-\varepsilon^{-\frac{R_7}{L_7}(t-t_8)}\right]-\frac{\sqrt{2}E_s}{Z_7}\left[\sin\left(\omega t+\frac{2}{3}\pi-\phi_7\right)-\varepsilon^{-\frac{R_7}{L_7}(t-t_8)}\sin\left(\omega t_8+\frac{2}{3}\pi-\phi_7\right)\right]+i_{d8}\,\varepsilon^{-\frac{R_7}{L_7}(t-t_8)}$$

$$24)\quad L_1=L_d+\frac{l_p}{2}+\frac{9}{24}L_c,\quad R_1=R_d+\frac{R_p}{2}+\frac{9}{24}R_c,\quad Z_1=\sqrt{\overline{\omega L_1}^2+\overline{R_1}^2},\quad \tan\phi_1=\frac{\omega L_1}{R_1}$$

$$25)\quad L_2=L_d+L_p+\frac{L_c}{2},\quad R_2=R_d+R_p+\frac{R_c}{2},\quad Z_2=\sqrt{\overline{\omega L_2}^2+\overline{R_2}^2},\quad \tan\phi_2=\frac{\omega L_2}{R_2}$$

$$26)\quad L_3=L_d+L_p+\frac{L_c}{3},\quad R_3=R_d+R_p+\frac{R_c}{3},\quad Z_3=\sqrt{\overline{\omega L_3}^2+\overline{R_3}^2},\quad \tan\phi_3=\frac{\omega L_3}{R_3}$$

$$27)\quad L_4=L_p+L_c,\quad R_4=R_p+R_c,\quad Z_4=\sqrt{\overline{\omega L_4}^2+\overline{R_4}^2},\quad \tan\phi_4=\frac{\omega L_4}{R_4}$$

$$28)\quad L_5=2L_p+L_c,\quad R_5=R_p+R_c,\quad Z_5=\sqrt{\overline{\omega L_5}^2+\overline{R_5}^2},\quad \tan\phi_5=\frac{\omega L_5}{R_5}$$

$$29)\quad L_6=L_d+\frac{L_c}{2},\quad R_6=R_d+\frac{R_p}{2}+\frac{R_c}{2},\quad Z_6=\sqrt{\overline{\omega L_6}^2+\overline{R_6}^2},\quad \tan\phi_6=\frac{\omega L_6}{R_6}$$

$$30)\quad L_7=L_d+L_p+L_c,\quad R_7=R_d+R_p+R_c,\quad Z_7=\sqrt{\overline{\omega L_7}^2+\overline{R_7}^2},\quad \tan\phi_7=\frac{\omega L_7}{R_7}$$