ON THE RADIAL COMPONENT OF LEAKAGE FLUX AND THE EDDY CURRENT LOSS ON SHIELD PLATES IN A TRANSFORMER WITH CYLINDRICAL-LAYER-WINDINGS

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I. INTRODUCTION

In core type transformer, the ampere-turn distribution in radial direction of the H.T. winding and those of the L.T. winding are arranged in a balanced condition so as to keep the mechanical force in the axial direction minimum. Fig. 1 in-

154 kV., we adopted Fig. 2 c with some vertical connecting bands as shown with dotted lines in the figure, (we may call it grid type), but unfortunately after 4 years service in the field, the transformer met with an accident to be short circuitted and the H. T. winding was burnt down. After disassembling the burnt winding, we found that the grid type

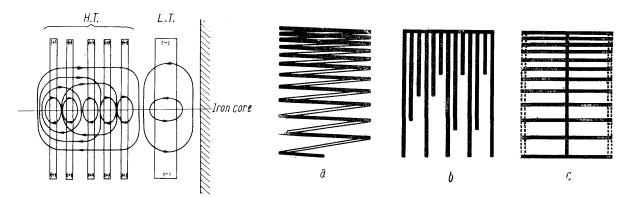


Fig. 1. The leakage flux

Fig. 2. The form of the shield plates

dicates the leakage flux of a transformer with cylindrical-layer-windings in this condition. The leakage flux consist of radial component and axial component but we can calculate the leakage impedance neglecting the former component, as long as the ampere-turns in the radial direction are in a balance.

In a transformer with cylindrical-layer-windings we apply static shields on the outer surface and inner surface of the H. T. winding so as to make it oscillationless and surge-proof. The form of the shield plates may either be a) spiral type, b) vertical curtain type or c) horizontal blind or rib type. (Fig. 2). The spiral type has a disadvantage that the voltage transmission from one end to the other takes some micro seconds when the no. of turns of the spiral increases. The vertical curtain type has some difficulty when it is prepared. We ordinary apply the horizontal blind type.

In our first practical product of oscillationless transformer, having a rating of 1 9,000 kVA.

outer and inner shields had long time been overheated by the cross current due to the radial component leakage flux. Taking out the burnt winding together with the L.T. winding in the air and exciting it with a low voltage corresponding to the short-circuit impedance of the transformer, we found that at the upper and lower ends of the winding the leakage flux in the radial direction amounts to ca 150 lines/cm² and the cross currents in the grid a little over a hundred amperes.

This small reports gives a method and the results of calculation of the radial component of leakage flux and the cross current or eddy current losses in the static shields of this type.

II. CALCULATION OF RADIAL COM-PONENT OF LEAKAGE FLUX

a) Calculation neglecting the effect of transformer core

In Fig. 3, the current I in length dl causes a

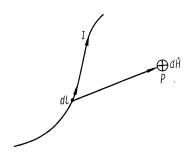


Fig. 3.

magnetic field strength $d\dot{H}$ at point P,

here \overrightarrow{r} is a distance vector between dl and P.

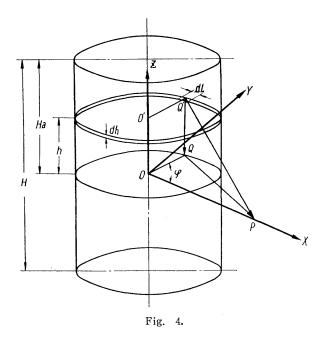


Fig. 4 is a cylindrical coil whose height is H and radius R. Let the total ampere turns of the coil be A (Effective value), then the magnetic field strength at point P will be calculated as follows. The amperes at point Q with a width of dh and a length of dl causes at P

$$d\vec{H} = \frac{\sqrt{2} A}{10H} dh \frac{\overrightarrow{dl} \times \overrightarrow{Q'P}}{\overrightarrow{Q'P}^3} \qquad \dots (2)$$

while

$$\overrightarrow{Q'P} = \overrightarrow{Q'Q} + \overrightarrow{QO} + \overrightarrow{OP}$$

therefore

$$d\vec{H} = \frac{\sqrt{2} A}{10 H} dh \frac{\overrightarrow{dl} \times (\overrightarrow{Q'Q} + \overrightarrow{QO} + \overrightarrow{OP})}{\overrightarrow{Q'P}^3} ... (2')$$

taking the unit vectors at point O as \dot{x} , \dot{y} and \dot{z}

$$\overrightarrow{dl} \times \overrightarrow{Q'Q} = Rd\varphi h \left(-\cos \varphi \dot{x} - \sin \varphi \dot{y} \right)$$

$$\overrightarrow{dl} \times \overrightarrow{OO} = Rd\varphi R \dot{z}$$

$$\overrightarrow{dl} imes \overrightarrow{OP} = -R d \varphi X \cos \varphi \dot{z}$$
 and $X = OP$

Indicating X-, Y-, and Z- component of $d\dot{H}$ by dH_x , dH_y and dH_z

$$\begin{split} dH_{x} &= -\frac{\sqrt{2}AR}{10H} \cdot \frac{h \ dh \cos \varphi \ d\varphi}{Q'P^{3}} \\ dH_{y} &= -\frac{\sqrt{2}AR}{10H} \cdot \frac{h \ dh \sin \varphi \ d\varphi}{Q'P^{3}} \\ dH_{z} &= -\frac{\sqrt{2}AR}{10H} \cdot \frac{dh(R - X \cos \varphi) \ d\varphi}{Q'P^{3}} \end{split}$$

while,

 $\overline{Q'P^2} = \overline{Q'Q^2} + \overline{QP^2} = h^2 + R^2 + X^2 - 2RX\cos\varphi$ the three components of magnetic field at P will thus be

$$H_{x} = -\frac{\sqrt{2}AR}{10H} \int_{-(H-Ha)}^{Ha} \int_{-(H-Ha)}^{2\pi} \frac{h \cos \varphi \, dh \, d\varphi}{(h^{2} + R^{2} + X^{2} - 2RX \cos \varphi)^{\frac{3}{2}}}$$

$$H_{y} = -\frac{\sqrt{2}AR}{10H} \int_{-(H-Ha)}^{Ha} \int_{-(H-Ha)}^{Ha} \frac{h \sin \varphi \, dh \, d\varphi}{(h^{2} + R^{2} + X^{2} - 2RX \cos \varphi)^{\frac{3}{2}}} = 0$$

$$H_{z} = \frac{\sqrt{2}AR}{10H} \int_{-(H-Ha)}^{Ha} \int_{-(H-Ha)}^{Ha} \frac{(R - X \cos \varphi) \, dh \, d\varphi}{(h^{2} + R^{2} + X^{2} - 2RX \cos \varphi)^{\frac{3}{2}}}$$

 H_x and H_z can be calculated in a similar way but here we need the value of the radial component H_x . In air or oil, the magnetic flux density coinsides with the magnetic field strength and the radial component of magnetic flux at P will be,

$$= \frac{2\sqrt{2}AR}{10H}$$

$$\left[\int_{0}^{\pi} \frac{\cos\varphi \,d\varphi}{(H_{a}^{2}+R^{2}+X^{2}-2RX\cos\varphi)^{\frac{1}{2}}} - \int_{0}^{\pi} \frac{\cos\varphi \,d\varphi}{\{(H-H_{a})^{2}+R^{2}+X^{2}-2RX\cos\varphi\}^{\frac{1}{2}}}\right]$$

$$= \frac{2\sqrt{2}A}{10H} \left\{\frac{H_{a}^{2}+R^{2}+X^{2}}{X} \cdot \frac{K(\alpha_{Ha})}{\sqrt{H_{a}^{2}+(R+X)^{2}}} - \frac{\sqrt{H_{a}^{2}+(R+X)^{2}}}{X}E(_{Ha}) - \frac{(H-H_{a})^{2}+R^{2}+X^{2}}{X} \times \frac{K(\alpha_{H-Ha})}{\sqrt{(H-H_{a})^{2}+(R+X)^{2}}} + \frac{\sqrt{(H-H_{a})^{2}+(R+X)^{2}}}{X}E(\alpha_{H-Ha})\right\}$$

$$= \frac{10H}{(M-H_{a})^{2}+(M-H_{a})^{2}} + \frac{(M-H_{a})^{2}+(M-H_{a})^{2}}{X}E(\alpha_{H-Ha})$$

$$= \frac{10H}{(M-H_{a})^{2}+(M-H_{a})^{2}} + \frac{(M-H_{a})^{2}+(M-H_{a})^{2}}{X}E(\alpha_{H-Ha})$$

$$= \frac{10H}{(M-H_{a})^{2}+(M-H_{a})^{2}} + \frac{(M-H_{a})^{2}+(M-H_{a})^{2}}{X}E(\alpha_{H-Ha})$$

$$= \frac{10H}{(M-H_{a})^{2}+(M-H_{a})^{2}} + \frac{(M-H_{a})^{2}+(M-H_{a})^{2}}{X}E(\alpha_{H-Ha})$$

Here $K(\alpha_H)$, $E(\alpha_H)$ are the first and the second kind complete elliptic integral and

$$\alpha_H = \sin^{-1} 2 \sqrt{\frac{RX}{H^2 + (R+X)^2}}$$
(5')

(As to the detailed calculation refer to the appendix).

b) Calculation taking account of the effect of transformer core

In case (a), the calculation was made neglecting the effect of transformer core in the coil. Fig. 5 shows a coil with a radius R having a core with a radius r within it. The transformer must have a shading effect. Let $PQ_{a'}$ and $PQ_{a''}$ in Fig. 5

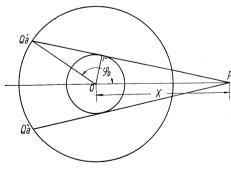


Fig. 5.

tangent lines from P and assuming that the effect of ampere turns in the arc $Q_a'Q_a''$ for P are negligibly small, the calculation will be as follows.

In case (a), integration on angle φ was made from $\varphi=0$ to $\varphi=\pi$ but now the same is made up to $\varphi=\angle POQ_a'=\varphi_a$. Putting r/R=m and r/X=n

$$\varphi_a = \cos^{-1} \{ mn - \sqrt{(1-m^2)(1-n^2)} \} \dots (6)$$

The result of calculation is

Here $F(\alpha, \phi)$ and $E(\alpha, \phi)$ are the first and the second kind elliptic integrals and $K(\alpha)$ and $E(\alpha)$ are the complete elliptic integrals. Further

$$\alpha_{H} = \sin^{-1} 2 \sqrt{\frac{RX}{H^{2} + (R + X)^{2}}}$$

$$\psi' = \frac{\pi}{2} - \frac{\varphi_{a}}{2}$$

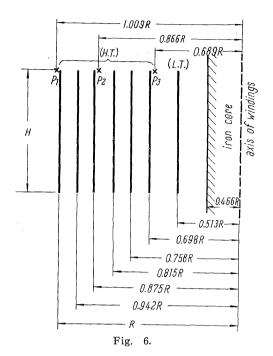
$$\psi'' = \tan^{-1} \left\{ \sqrt{\frac{H^{2} + (R + X)^{2}}{H^{2} + (R - X)^{2}}} \tan \frac{\varphi_{a}}{2} \right\} \dots (7')$$

For a number of concentric coils, by taking their respective radius R and ampere turns A and by calculating by equation (5) or (7) and summing them up, the radial component of magnetic flux at P can be obtained. In the transformer, of course, the ampere turns of the H.T. winding and that of the L.T. winding take different signs (plus and minus).

III. EXAMPLE OF THE FLUX DENSITY IN RADIAL DIRECTION

a) Example of Calculation for $H_a=0$.

Fig. 6 is section of a transformer whose H. T. winding consists of 6 coils and L. T. winding being concentrated in one coil. The diameter of each coil is indicated in the figure and the ampere turns of each coil is as followings, taking those of the L. T. coil as -A.



		L. T. winding				
1st	2nd	3rd	4th	5th	6th	7th coil
coil	coil	coil	coil	coil	coil	
A	A	A	A	A	A	- A
0.1433	0.1942	0.1308	0.1942	0.1942	0.1433	

Putting equation (5) or (7) in the form $B_x = \frac{2\sqrt{2}A}{10H}C$, the calculated values of C for point P_1 , P_2 and P_3 in the figure are

For P_1

	1st coil	2nd coil	3rd coil	4th coil	5th coil	6th coil	7th coil
From (5)	3.907	2.425	1.800	1.478	1.172	0.938	0.418
From (7)	3.720	2.605	1.923	1.540	1.255	1.020	0.582

For P_2

	1st coil	2nd coil	3rd coil	4th coil	5th coil	6th coil	7th coil
From (5)	2.107	2.567	3.908	2.618	1.911	1.472	0.622
From (7)	2.065	2.691	3.816	2.645	2.077	1.620	0.795

For P_3

	1st coil	2nd coil	3rd coil	4th coil	5th coil	6th coil	7th coil
From (5)	1.330	1.432	1.635	1.900	2.380	3.674	0.952
From (7)	1.549	1.670	1.883	2.170	2.545	3.595	1.437

Multiplying above values of C by $2\sqrt{2}A/10H$ and summing up, maximum flux densities at P_1 , P_2 and P_3 are obtained as follows.

	at P ₁	at P_2	at P_3
From (5)	196	223	139
From (7)	183	219	103

The flux density at P_2 is larger than that at P_1 or P_3 because P_2 is in the midpoint of H. T. winding, while that at P_3 is smaller than that of P_1 because L. T. winding (minus ampere turns) is close near P_3 .

Above mentioned transformer of our first oscillationless transformer was fitted with grid type static shields at P_1 , P_2 and P_3 and the fact that degree of burning or damage after 4 years running was maximum on the shield at P_2 and minimum on that at P_3 coinsides with the values of the calculated flux densities.

(b) Flux distribution along the axis of winding

By changing the value of H_a in (5) and (7), the flux distribution at radius X along the axis of winding will be obtained and after calculating the values for a few points for a fixed X, I tried to denote the flux distribution curve by following formula.

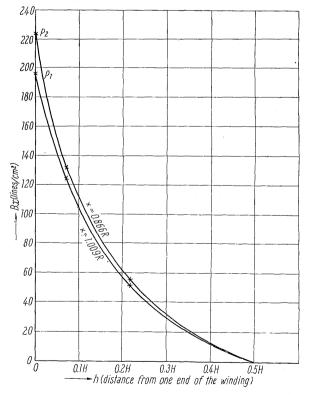


Fig. 7.

Here h = d

 $B_x = B_0 - B' \log (K + h)$ h =distance from one end of the winding B_0 , B', K; constant

$$B_0 = B' \log \left(K + \frac{H}{2}\right)$$

H=height of the winding

The flux density is zero at $H_a = H/2$

For the transformer shown in Fig. 6, examples of this distribution curve corresponding the radius P_1 and P_2 according to calculations by equation (5) are given in Fig. 7. These two curves are respectively indicated by for the cylindrical surface including P_1

$$B_x = 238.3 - 127 \log (6 + h)$$
(9)

for the cylindrical surface including P_2 .

$$B_x = 340 - 170 \log (4.5 + h)....(10)$$

IV. EDDY CURRENT LOSSES IN THE STATIC SHIELDS

(a) Eddy currents in a thin horizontal band conductor (endless but not making one turn)

Eddy currents caused by radial component magnetic flux are expressed by

$$\frac{di}{dh} = \omega \sigma B_x \qquad \dots (11)$$

Here

$$\omega = 2\pi f$$
 $f = \text{frequency}$
 $\sigma = \text{conductivity of the material}$

From equation (8)

$$i = \sigma \sigma \int \{B_0 - B' \log (K+h)\} dh$$

$$= \sigma \sigma \{B_0 h - B' (K+h) \log (K+h) + B' (K+h) M+C\} \dots (12)$$

$$M = 0.4343 \qquad C = \text{constant}$$

denoting σ by \mathfrak{F}/cm and i by A/cm^2

$$i = \frac{\omega\sigma}{\sqrt{2}} \left\{ B_0 h - B'(K+h) \log (K+h) + 0.434 B'(K+h) + C \right\} 10^{-8}$$

The value of C can be determined by putting the total sum of the currents in the horizontal direction zero. Let the width of the band h_2-h_1 , that is, the band lie between $h=h_1$ and $h=h_2$ and its thickness t

$$t\int_{h}^{h_2} idh = 0$$

From (12) we get

$$\begin{vmatrix}
B_0h^2 - B'(K+h)^2 \left\{ \log(K+h) - 0.6515 \right\} \\
+ 2Ch \begin{vmatrix}
h^2 \\
h_1
\end{vmatrix} = 0$$
(13)

which gives the value of C and finally the eddy current loss in unit length is given by

$$W = \frac{1}{\sigma} t \int_{h}^{h_2} i^2 dh \qquad \dots (14)$$

(b) Eddy current losses when the ends of bands are closed

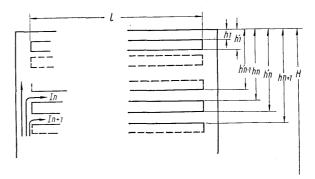


Fig. 8.

When the ends of the bands are closed as shown in Fig. 8, the values of constants C are obtained in following way. The figure corresponds to $\frac{1}{2}$ of total height H of the coil and (N+1) of horizontal bands are closed at their both ends.

Let the resistance of the horizontal parts, length l, of the nth bands be R_n and that of the vertical part between nth and (n+1)th be r_n . The voltage caused by the magnetic flux in the gap between nth and (n+1)th bands is

$$\omega l \int_{hn}^{h'n} B_x dh$$

and the total current in the nth band is

$$I_n = t \int_{h'n-1}^{hn} i \, dh = \omega \sigma t \int_{h'n-1}^{hn}$$

Therefore for the nth mesh circuit.

$$\omega l \int_{hn}^{h'n} B_x dh = C_n R_n - C_{n+1} R_{n+1}$$

$$-2r_n \omega C t \sum_{n=1}^n \int_{h'n-1}^{hn} \int_{h} B_x dh \ dh$$
.....(15)

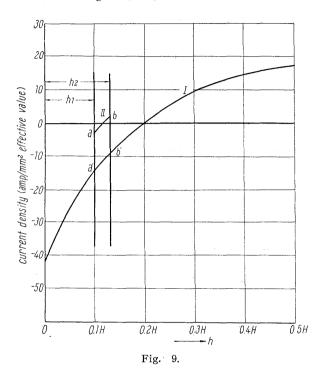
here C_n is the integral constant for $\int B_x dh$ of nth band.

From equation (15), N equations are obtained corresponding to $n=1, 2, \ldots, n$, and thus the values for c_1, c_2, \ldots, c_N , c_{N+1} may be obtained from these N equations and

$$\sum_{n=1}^{n=N+1} \int_{h'n-1}^{hn} \int_{h'n-1} B_x dh \, dh = 0 \qquad \dots (16)$$

(c) Examples of calculations

Fig. 9 gives some examples of calculations of eddy current losses. The curve I for a sheet of copper plate of height H at the radius corresponding to P_2 and the curve II for a band of the same material of height h_2-h_1 at same radius and $h=h_1$.



The calculations were made with B_x , i and C respectively from equations (10), (12) and (13). The curve ab is point to point parallel with a'b'.

V. CONCLUSIONS

The grid type static shields applied on our first practical oscillationless transformers, unfortunately burnt down after 4 years service on site, necessitated us to axamine its reason of overheating. The radial component of the leakage flux which at first was quite unknown was calculated by applying the law of Biot-Savart and using the elliptic integrals, taking account of the shielding effect of the magnetic core. The magnitude of leakage flux was found to be ca 200 lines/cm² at its maximum point and corresponding eddy current losses hereby calculated suggested us to use the horizontal blind or rib type shields with horizontal bands of adequate widths.

APPENDIX

The calculation of equation (4) in the text can be made, by putting

$$a = H^2 + R^2 + X^2$$
$$b = 2RX$$

and solving

$$Y = \int_{0}^{\varphi_{a}} \frac{\cos \varphi}{(a - b \cos \varphi)^{\frac{1}{2}}} d\varphi \qquad \dots \dots (\text{App. 1})$$
Putting
$$\tan \varphi/2 = x$$

$$\frac{dx}{d\varphi} = \frac{1}{2 \cos^{2} \frac{\varphi}{2}}$$

$$d\varphi = 2 \cos^{2} \frac{\varphi}{2} dx = \frac{2}{1 + x^{2}} dx$$

$$\cos \varphi = 2 \cos^{2} \frac{\varphi}{2} - 1 = \frac{2}{1 + x^{2}} - 1$$
therefore
$$Y = \int_{0}^{x = \tan^{\frac{\varphi_{a}}{2}}} \frac{4x dx}{(1 + x^{2}) \sqrt{(1 + x^{2})} \{(a + b)(1 + x^{2}) - 2b\}}$$

$$- \int_{0}^{x = \tan^{\frac{\varphi_{a}}{2}}} \frac{2dx}{\sqrt{(1 + x^{2})} \{(a + b)(1 + x^{2}) - 2b\}}$$

$$- \int_{0}^{x = \tan^{\frac{\varphi_{a}}{2}}} \frac{dx}{\sqrt{(1 + x^{2})} \{(a + b)(1 + x^{2}) - 2b\}}$$

$$= \frac{4}{\sqrt{a + b}} \int_{0}^{x = \tan^{\frac{\varphi_{a}}{2}}} \frac{dx}{\sqrt{(1 + x^{2})^{3}} (\frac{a - b}{a + b} + x^{2})}$$

$$= \frac{4}{\sqrt{a + b}} \int_{0}^{x = a - \frac{\varphi_{a}}{2}} \frac{dx}{\sqrt{(1 + x^{2})^{3}} (\frac{a - b}{a + b} + x^{2})}$$

$$= \frac{4}{\sqrt{a + b}} \int_{0}^{x = a - \frac{\varphi_{a}}{2}} \frac{dx}{\sqrt{(1 + x^{2})^{3}} (\frac{a - b}{a + b} + x^{2})}$$

$$= \frac{4}{\sqrt{a + b}} \int_{0}^{x = a - \frac{\varphi_{a}}{2}} \frac{dx}{\sqrt{(1 + x^{2})^{3}} (\frac{a - b}{a + b} + x^{2})}$$

$$= \frac{4}{\sqrt{a + b}} \left\{ K(\alpha) - E(\alpha) - F(\alpha, \psi') + E(\alpha, \psi') \right\}$$

$$= \frac{2\sqrt{a + b}}{b} \left\{ K(\alpha) - E(\alpha) - F(\alpha, \psi') + E(\alpha, \psi') \right\}$$

$$= \frac{1}{x} = \frac{1}{\tan^{\frac{\varphi_{a}}{2}}} = \cot^{\frac{\varphi_{a}}{2}}$$

$$\sin^{2} \alpha = 1 - \frac{a - b}{a + b} = \frac{2b}{a + b}$$

$$2nd \ \text{term} = \frac{2}{\sqrt{a + b}} \int_{0}^{x = \tan^{\frac{\varphi_{a}}{2}}} \frac{dx}{\sqrt{(1 + x^{2})(\frac{a - b}{a + b} + x^{2})}}$$

 $= \frac{2}{\sqrt{a+b}} F(\alpha, \phi'') \dots (App. 4)$

where

$$\tan \psi'' = x \sqrt{\frac{a+b}{a-b}} = \sqrt{\frac{a+b}{a-b}} \tan \frac{\varphi_a}{2} \dots \dots$$

$$\sin^2 \alpha = 1 - \frac{a-b}{a+b} = \frac{2b}{a+b} \tag{App. 4'}$$

therefore

$$\mathbf{Y} = \frac{2\sqrt{a+b}}{b} \{ K(\alpha) - E(\alpha) - F(\alpha, \psi') + E(\alpha, \psi') \} - \frac{2}{\sqrt{a+b}} F(\alpha, \psi'')$$
......(App. 5)

when
$$\varphi_a = \pi$$
 then $\tan \varphi' = 0$, $\tan \varphi'' = \infty$

$$(\mathbf{Y})_{\varphi_a = \pi} = \frac{2\sqrt{a+b}}{b} \{K(\alpha) - E(\alpha)\}$$

$$-\frac{2}{\sqrt{a+b}} K(\alpha)$$

$$= \frac{2a}{b\sqrt{a+b}} K(\alpha) - \frac{2\sqrt{a+b}}{b} E(\alpha)$$
.......(App. 6)

OIL-RESISTANT RUBBER PACKING

Ву

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I. FOREWORD

Packing is used everywhere not only in electrial machines or chemical apparatus but also in articles of daily use. We know "packing" is so familiar when we remember in our home that bottle for medicine or cosmetics has packing made of rubber or vinylresin inside its cover preventing inner substance to leak. We find its important service when we take a cup of beer and imagine if a cork packing within the lid of container becomes useless, how untasteful the drink would be.

In the same manner, packings are indispensable in the electrical machine, and it is difficult to find any machine without packing. In spite of its importance, the research about the functionality and material of packing had been apt to be forgotten for the reason of its non-attractive work. But to-day higher grade characters are required about electrical machines, and the research has been making a progress to know how to use and what kind of material should be used for packing, and recently we can find some reports about packing in bulletins of many countries.

In this article I will explain about characters of the oil-resistant rubber packing, utilized for transformer or other oil-immersed type electrical machines, quoting our experiments about its elastic behavior.

II. CHARACTERS NECESSARY FOR THE OIL-RESISTANT RUBBER PACKING

Oil-resistant rubber packing employed in electrical machine should have many characters, although some of these are not always necessary for one machine or one characteristic is especially important for other machine.

Necessary characters are as follows:

1. Low permanent compression set

Permanent compression set is defined as plastic deformation of packing which occurs when packing is used for a long time. The stress, necessary for initial strain, decreases as time elapses. The worst happens when the stress becomes zero owing to perfect plastic deformation and functionality of packing is completely lost. The contrast to this case is that when the compressive stress keeps its initial value. This is when packing is perfectly elastic. Actual packing cannot be perfectly elastic, and so we are obliged to say that the more elastic, the better. For this reason low permanent compression set is desirable not only for the oil-resistant type but also for the others in general.

2. Low swelling when immersed in insulating

Large swelling of rubber packing results in the loss of functionality of packing owing to slip out of the packing, and in an extreme case, destruction of machine occurs by swelling pressure. In physicochemical sense the oil resistance of rubber is that rubber does not swell and lose its mechanical strength, owing to the diffusion or permeation of oil molecules in rubber. Accordingly, low swelling is indispensable as rubber packing. But as the measurement of swelling is rather difficult, weight increase by immersion in oil is measured for convenience as